[This question paper contains 4 printed pages.]

6591 Your Roll No. .....

B

B.Sc. (Hons.) COMPUTER SCIENCE / III Semester

Paper – CS 301 : ALGORITHMS

(Admissions of 2001 to 2009)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*

Parts of a question should be attempted together.

1. (a) Find the LV decomposition of the given matrix :

$$\begin{pmatrix} 1 & -8 & 6 \\ 10 & 3 & -4 \\ 8 & 5 & 2 \end{pmatrix} \quad (4)$$

(b) Consider the given dynamic-programming algorithm to compute a binomial coefficient

$$C(n, K) = \frac{n!}{K!(n-K)!}$$

BINOMIAL (n, K)

// Input : A pair of non negative integers

$$n \geq K \geq 0$$

// Output : The value of C(n, K)

1. for i ← 0 to n do

2. for j ← 0 to min (i, K) do

3. if (j = 0) or (j = K)

4. C[i, j] ← 1

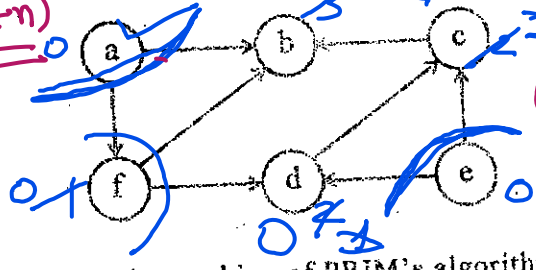
5. else C[i, j] ← C[i–1, j–1] + C[i–1, j]

6. return C[n, K]

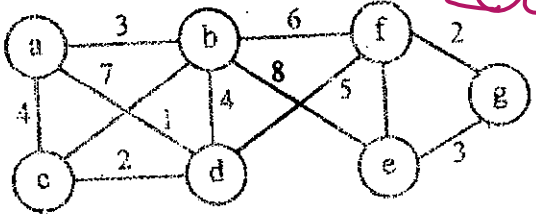
Compute the worst-case running time for the algorithm. (4)

(c) Give a dynamic-programming algorithm to solve the 0–1 knapsack problem. (6)

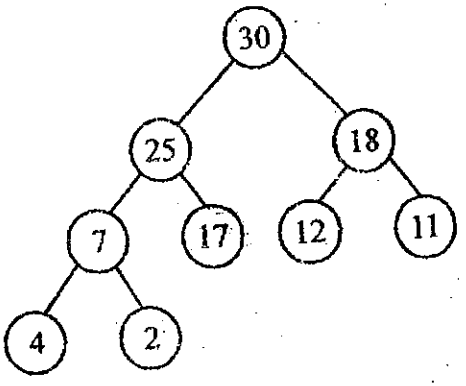
2. (a) Show the ordering of vertices produced by TOPOLOGICAL – SORT when it is run on the given directed acyclic graph. Consider the vertices in alphabetical order. (5)



(b) Illustrate the working of PRIM's algorithm on the given graph : (5)



3. (a) Insert 43 followed by 20 into following Max Heap.



(4)

(b) Prove that the worst-case running time of RANDOMIZED–QUICKSORT is $O(n^2)$. (5)

(c) Illustrate the operation of BUCKET–SORT on the array :

A = <·90, ·21, ·19, ·81, ·38, ·73, ·41, ·70, ·59, ·04> (4)

(d) Show the working of the SELECT(i) algorithm (that finds the i^{th} order statistic in worst-case linear time) on the given array for i = 5 :

A = <41, 20, 8, 64, 3, 81, 60> (4)

4. (a) Show the step-by-step construction of BINOMIAL–HEAP when the nodes 24, 48, 32, 64, 12, 91, 83 are successively inserted into an initially empty binomial heap. (5)

(b) Show the construction of the red-black tree after successively adding the keys 39, 83, 42, 94, 61 into an initially empty tree. (5)

(c) Show how the dynamic set query MINIMUM can be supported in O(1) worst-case time on an augmented OS-tree. (3)

5. (a) Determine the cost and structure of an optimal binary search tree for a set of n = 5 keys, with the following probabilities :

i	0	1	2	3	4	5
p_i	–	0.04	0.06	0.08	0.02	0.10
q_i	0.06	0.06	0.06	0.05	0.05	0.05

(6)

(b) Consider the STACK data structure that implements PUSH, POP and MULTIPOP operations. Use POTENTIAL METHOD to show that the amortized cost for a sequence of n PUSH, POP and MULTIPOP operations is O(n). (4)

(c) What is memoization ? How it improves the performance of a pure recursive algorithm ? Explain with an example. (6)

6. Coins of denominations Rs. 1, 4 and 5 are available. A person should pick up minimum number of coins so that the total currency he has is Rs. 8. Give an efficient algorithm to find an optimal solution for this problem. Explain with an example.