FRACTALS 00 STATISTICAL FRACTALS

ANALYSIS

◆□▶ ◆□▶ ◆ □▶ ◆ □▶ ○ □ ○ ○ ○ ○

Statistics and Computer Application GENERATION AND ANALYSIS OF STATISTICAL FRACTALS

Bidisha Bandyopadhyay

Department of Physics and Astrophysics University of Delhi

FRACTALS oo	STATISTICAL FRACTALS	ANALYSIS 000000	CONCLUSION

▲日▼▲□▼▲□▼▲□▼ □ ののの

Outline

FRACTALS

- Types of Fractals
- Fractal Dimension

2 STATISTICAL FRACTALS

- Levy Flight
- Generation Of Levy Flight

3 ANALYSIS

- The Cumulative Distribution
- The Methods
 - Asymptotic Method
 - Fourier v/s Asymptotic
- Determining Fractal Dimension

4 CONCLUSION

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION

◆□▶ ▲□▶ ▲目▶ ▲目▶ ▲□▶

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00	0000	000000	

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00	0000	000000	

• SELF-SIMILAR structures



RACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION

◆□▶ ◆□▶ ◆三▶ ◆三▶ - 三 - のへぐ

- SELF-SIMILAR structures
- Characteristic FRACTAL DIMENSION

FRACTALS	STATISTICAL FRACTALS	ANALYSIS
00	0000	000000

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

- SELF-SIMILAR structures
- Characteristic FRACTAL DIMENSION
- Not DIFFERENTIABLE anywhere

FRAC	TALS
00	

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

What are FRACTALS?

- SELF-SIMILAR structures
- Characteristic FRACTAL DIMENSION
- Not DIFFERENTIABLE anywhere



Reference (Google Images)

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
0			
Types of Fractals			

◆□▶ ▲□▶ ▲目▶ ▲目▶ ▲□▶

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
0			
Types of Fractals			

Types of FRACTALS

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLU
0			
Types of Fractals			

Types of FRACTALS

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

• Deterministic Fractal (*e.g.* Cantor Set, Mandelbrot Set, Koch Curve)

FRACTALS ●○ Types of Fractals STATISTICAL FRACTALS

ANALYSIS 000000 CONCLUSION

◆ロ > ◆母 > ◆臣 > ◆臣 > ○日 ○ ○ ○ ○

Types of FRACTALS

• Deterministic Fractal (*e.g.* Cantor Set, Mandelbrot Set, Koch Curve)



FRACTA	LS
••	
Types of	Fractals

ANALYSIS 000000

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

Types of FRACTALS

• Deterministic Fractal (*e.g.* Cantor Set, Mandelbrot Set, Koch Curve)



• Statistical Fractal (e.g. Levy Flight)

FRACTA	LS
••	
Types of	Fractals

ANALYSIS 000000

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

Types of FRACTALS

• Deterministic Fractal (*e.g.* Cantor Set, Mandelbrot Set, Koch Curve)



• Statistical Fractal (e.g. Levy Flight)



FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00			
Fractal Dimension			

▲ロト ▲園 ▶ ▲ 国 ▶ ▲ 国 ▶ の Q ()

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00			
Fractal Dimension			

How to determine Fractal Dimension?

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00			
Fractal Dimension			

How to determine Fractal Dimension?

Three methods :



FRAC	TALS
00	

ANALYSIS 000000

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

Fractal Dimension

How to determine Fractal Dimension?

Three methods :



FRACT	ALS
00	

ANALYSIS 000000

Fractal Dimension

How to determine Fractal Dimension?

Three methods :



$$a = \frac{1}{s^D}$$

Box Counting Method

$$D = \frac{\log(\text{No. of Boxes containing points})}{\log(\text{Scale of Box})}$$

FRACT	ALS
00	
Fractal	Dimension

ANALYSIS 000000

How to determine Fractal Dimension?

Three methods :



$$a = \frac{1}{s^D}$$

Box Counting Method

$$D = \frac{\log(\text{No. of Boxes containing points})}{\log(\text{Scale of Box})}$$

General Method

$$D = \frac{\log(No.ofpoints)}{\log(Radius)}$$

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

▲ロト ▲園 ▶ ▲ 国 ▶ ▲ 国 ▶ の Q ()

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

• Markovian Stochastic Processes



FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

• Markovian Stochastic Processes

• PDF follow asymptotic *Power Law*

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

- Markovian Stochastic Processes
- PDF follow asymptotic *Power Law*
- Diverging Variance

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

▲日▼▲□▼▲□▼▲□▼ □ ののの

- Markovian Stochastic Processes
- PDF follow asymptotic *Power Law*
- Diverging Variance
- Statistically Self Repeatating (Fractal Dimension= α)

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

▲ロト ▲園 ▶ ▲ 国 ▶ ▲ 国 ▶ の Q ()

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

Asymptotic Behaviour

 $\lambda(x) \simeq |x|^{-1-\alpha}$

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

Asymptotic Behaviour

$$\lambda(x) \simeq |x|^{-1-\alpha}$$

Behaviour in Fourier Space

$$f(k) = \exp\left[-i\mu k - \sigma^{\alpha}|k|^{\alpha} \left(1 - i\beta \frac{k}{|k|} \varpi(k, \alpha)\right)\right]$$
$$\varpi = \begin{cases} \tan \frac{\pi \alpha}{2} & \text{if } \alpha \neq 1, 0 < \alpha < 2\\ -\frac{2}{\pi} \ln |k| & \text{if } \alpha = 1 \end{cases}$$
$$f(k) = \exp\left[-|k|^{\alpha}\right]$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ - 三 - のへぐ

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			

▲ロト ▲園 ▶ ▲ 国 ▶ ▲ 国 ▶ の Q ()

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Levy Flight			



・ロト ・日・ ・ ヨ・

æ

f(x) v/s x

Fri May 04 12:30:08 2012

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Generation Of Levy Flight			

▲□▶ ▲□▶ ▲三▶ ▲三▶ ▲□ ● ● ●

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Generation Of Levy Flight			

How to Generate A Levy Flight Distribution?

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Generation Of Levy Flight			

How to Generate A Levy Flight Distribution?

◆□▶ ◆□▶ ◆目▶ ◆目▶ ▲□ ◆ ��や

• Asymptotic Method

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
	0000		
Generation Of Levy Flight			

How to Generate A Levy Flight Distribution?

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● ○○

- Asymptotic Method
- Fourier Method

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
		00000	
The Cumulative Distribution			

▲ロト ▲御 ▶ ▲ 臣 ▶ ▲ 臣 ▶ ○ 臣 = の Q ()

FRACTALS 00	STATISTICAL FRACTALS	ANALYSIS •••••	CONCLUSION
The Cumulative Distribution			
	CX		
	$F(x) = \int_{-\infty}^{\infty} f$	(x)dx	
	J_0		J

▲□▶ ▲圖▶ ▲≣▶ ▲≣▶ 三回 - のへの

FRACTALS	
00	

ANALYSIS 000000

The Cumulative Distribution





FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
		00000	
The Methods			

・ロト・日本・日本・日本・日本

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
		00000	
The Methods			

Asymptotic Method

$$f(x) = A \begin{cases} 1 & 0 < x < 1 \\ \frac{1}{x^{-1-\alpha}} & x > 1 \end{cases}$$
$$A = \frac{1+\alpha}{\alpha}$$

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
		00000	
The Methods			

Asymptotic Method

$$f(x) = A \begin{cases} 1 & 0 < x < 1 \\ \frac{1}{x^{-1-\alpha}} & x > 1 \end{cases}$$
$$A = \frac{1+\alpha}{\alpha}$$

Fourier Method

$$f(k) = \exp\left[-|k|^{\alpha}\right]$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ - 三 - のへぐ

ANALYSIS

The Methods



Figure: Levy Distribution with lpha=1.2 at different scales as $\alpha < 0.00$

FRACTALS

STATISTICAL FRACTALS

ANALYSIS

The Methods



▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●





Figure: Comparison between Asymptotic Method and Fourier Method

・ロト ・ 日 ト ・ モ ト ・ モ ト

E 996

FRACTALS 00 STATISTICAL FRACTALS

ANALYSIS

Determining Fractal Dimension



◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 - ���?

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00	0000	000000	

• Wide Area of Applications (*e.g.*Structure Formation, Spread of Disease, *etc.*)

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00	0000	000000	

• Wide Area of Applications (*e.g.*Structure Formation, Spread of Disease, *etc.*)

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

• Our result gives $\alpha = 1.42$ with relative error 18.33%.

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSIO
00	0000	000000	

• Wide Area of Applications (*e.g.*Structure Formation, Spread of Disease, *etc.*)

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

- Our result gives $\alpha = 1.42$ with relative error 18.33%.
- Numerical Methods have their limitation.

FRACTALS	STATISTICAL FRACTALS	ANALYSIS

▲ロ ▶ ▲周 ▶ ▲ 国 ▶ ▲ 国 ▶ ● の Q @

CONCLUSION

- Wide Area of Applications (*e.g.*Structure Formation, Spread of Disease, *etc.*)
- Our result gives $\alpha = 1.42$ with relative error 18.33%.
- Numerical Methods have their limitation.
- Further scope for improvement in Numerical Techniques

FRACTALS 00

◆ロ > ◆母 > ◆臣 > ◆臣 > ○日 ○ ○ ○ ○

ACKNOWLEDGEMENT

I would like to thank Prof.T.R.Seshadri and Dr. Poonam Mehta for their constant support and guidance in this work.

FRACTALS	STATISTICAL FRACTALS	ANALYSIS	CONCLUSION
00	0000	000000	

THANK YOU