Statistics and Computer Application

GENERATION AND ANALYSIS OF STATISTICAL FRACTALS

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Outline

What are FRACTALS?

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- SELF-SIMILAR structures

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- Characteristic FRACTAL DIMENSION

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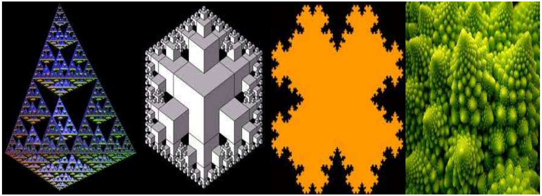

Reference (Google Images)

Types of Fractals

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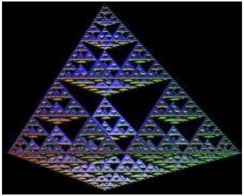

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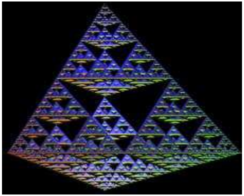

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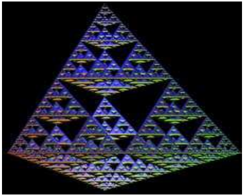

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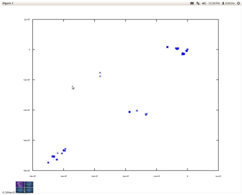

Fractal Dimension

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Self-Similarity Method

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General Method

$$D = \frac{\log(No.ofpoints)}{\log(Radius)}$$

Levy Flight

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- Statistically *Self Repeatating* (Fractal Dimension$=\alpha$)

Levy Flight

Asymptotic Behaviour

$$\lambda(x) \simeq |x|^{-1-\alpha}$$

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Behaviour in Fourier Space

$$\begin{aligned} f(k) &= \exp\left[-i\mu k - \sigma^{\alpha}|k|^{\alpha}\left(1 - i\beta\frac{k}{|k|}\varpi(k,\alpha)\right)\right] \\ \varpi &= \begin{cases} \tan\frac{\pi\alpha}{2} & \text{if } \alpha \neq 1, 0 < \alpha < 2 \\ -\frac{2}{\pi} ln|k| & \text{if } \alpha = 1 \end{cases} \\ f(k) &= \exp\left[-|k|^{\alpha}\right] \end{aligned}$$

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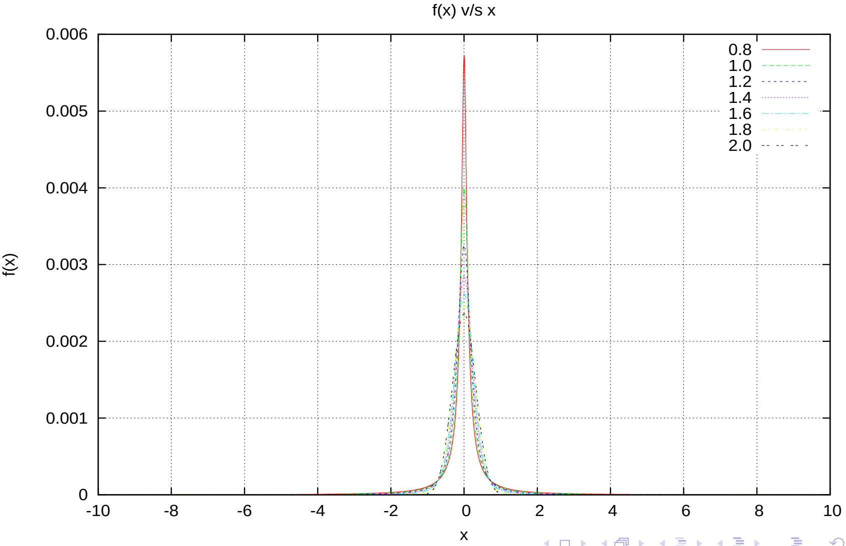

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Generation Of Levy Flight

How to Generate A Levy Flight Distribution?

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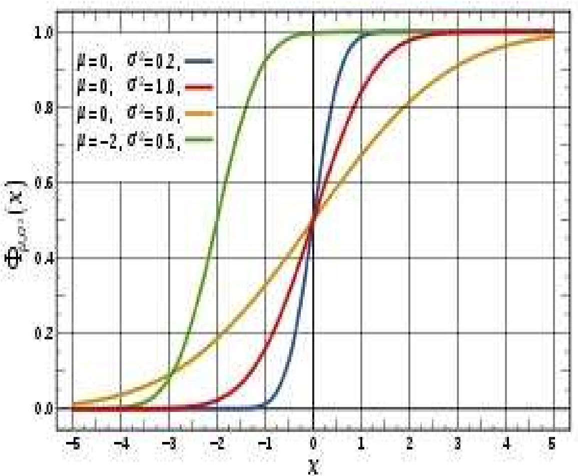

The Methods

Asymptotic Method

$$f(x) = A \begin{cases} 1 & 0 < x < 1 \\ \frac{1}{x^{-1-\alpha}} & x > 1 \end{cases}$$

$$A = \frac{1+\alpha}{\alpha}$$

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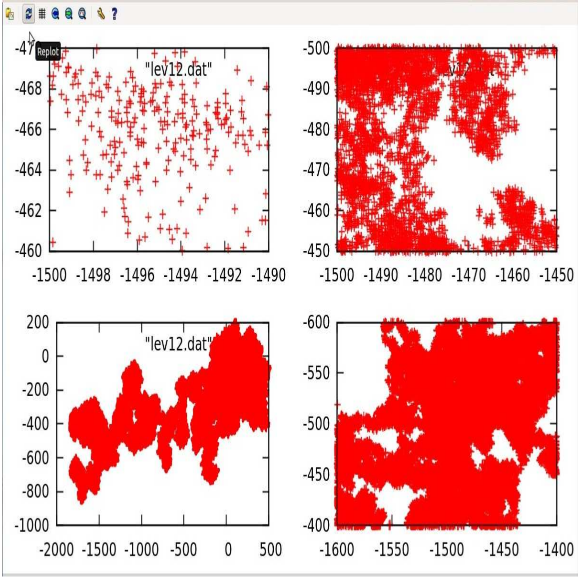
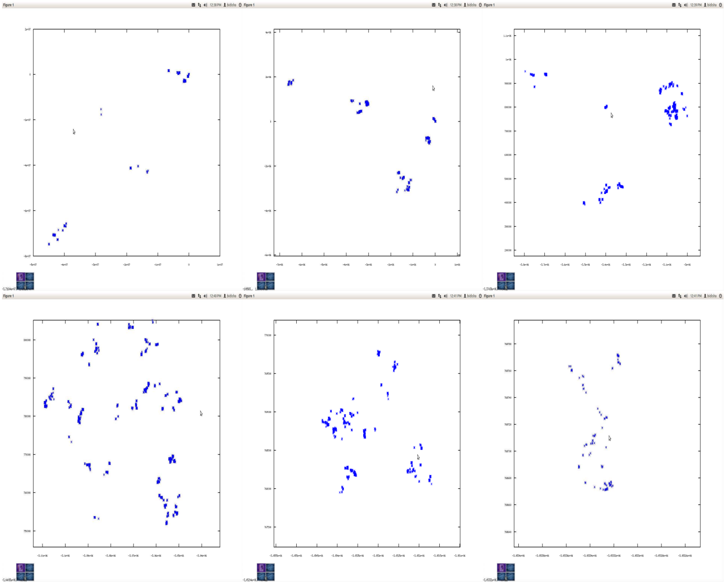


Figure: Levy Distribution with $\alpha = 1.2$ at different scales

The Methods



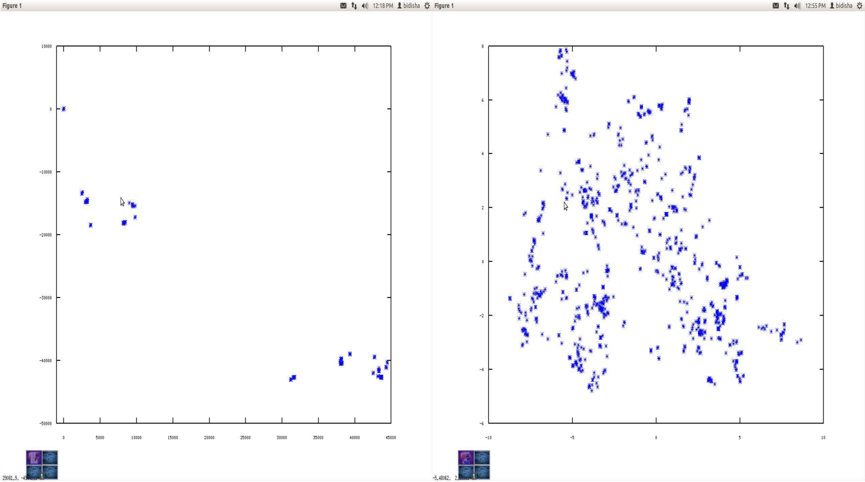
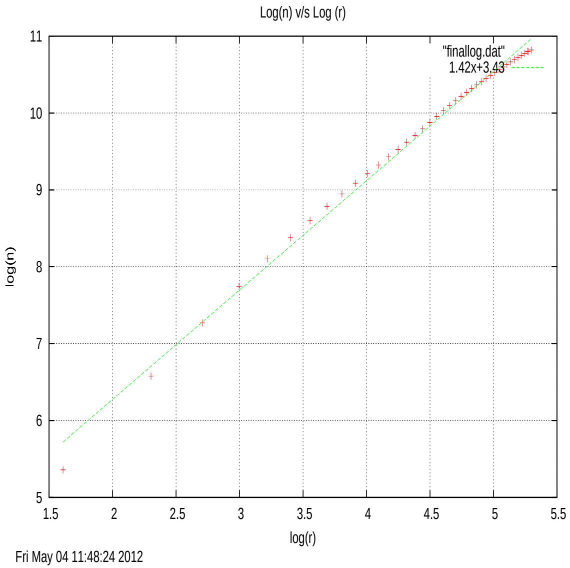


Figure: Comparison between Asymptotic Method and Fourier Method

Determining Fractal Dimension



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- Our result gives $\alpha = 1.42$ with relative error 18.33%.
- Numerical Methods have their limitation.
- Further scope for improvement in Numerical Techniques

ACKNOWLEDGEMENT

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THANK YOU