A Report

on

# "An Overview of Big Bang Nucleosynthesis Codes"

By

Rohin Kumar

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under the guidance of

Prof. Daksh Lohiya

Department of Physics & Astrophysics

University of Delhi
Delhi-110007

## Contents

# 1 Introduction

Big Bang Nucleosynthesis is one of the successes of the standard model of Cosmology. The original work done by Gamow & Ralph Alpher predicted the right amount of Helium and even cosmic microwave background radiation before it was experimentally verified. Owing to the complexity of the calculations involved, the nucleosynthesis process is to be numerically simulated. This report gives an overview of the algorithm behind the nucleosynthesis codes (mainly the original Kawano code). The simulations are performed for standard values and results are presented.

The following report is divided into sections. After this introduction, the second section talks about the basics of nucleosynthesis and the difference between stellar and big bang nucleosynthesis. The third section starts with the thermal history of the universe and goes on to discuss the fundamental interactions in the early universe leading to the expansion and further processes in BBN. The fourth section focuses on the Helium nucleosynthesis and the basic mathematics/algorithm that forms the crux of BBN codes. The fifth section gives the basic components of a standard BBN code. The simulation parameters to be set for run are briefed. The Results and conclusions are presented in the subsequent sections.

# 2 Nucleosynthesis

Nucleosynthesis is the process of formation of nucleus. For any nuclear reaction to take place a particle of charge has to penetrate the electrostatic repulsion. For example, if nuclei of charge $Z_1$ and $Z_2$ have to come closer to a distance $r$ we need to overcome the coloumb barrier.

$$V = \frac{Z_1 Z_2 e^2}{r} = \frac{1.44 Z_1 Z_2}{r(fm)}\ MeV$$

The average thermal energy of particles in the Maxwell-Boltzmann distribution is

$$kT = 8.62 \times 10^{-8} T\ keV$$

As it evident from the energy needed from this equation we need the temperatures of the order of $10^6$ or higher to cross this barrier for the nuclear reaction to take place. We know that this order of temperatures exist typically in the stars. Gamow, first developed the theory of alpha decay that gives us an idea (The penetration factor is proportional to $exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right)$ ) of the order of cross-section involved in the nuclear reactions. Later this idea was extended to develop the theory for Big Bang Nucleosynthesis.

## 2.1 Stellar vs. Primordial

There is a remarkable difference between the nucleosynthesis process that takes place in stars and the one that happens in the early universe right after the big bang. The time scale available for nucleosynthesis in stars is typically of the order of billions of years while for big bang nucleosynthesis the time scale is of the order of minutes. The stellar nucleosynthesis process is like isothermic process where as primordial nucleosynthesis is an adiabatic process of rapid cooling. The density in the stars is of the order of $100g/cm^3$ where as the density in the big bang conditions is as low as $10^{-5}$. The photon-to-baryon ratio which is a very important parameter in the chain of nuclear reactions that effects photo-disassociation in stellar nucleosynthesis is less than 1 photon per baryon but at big bang conditions there are billions of photons available for a single baryon. These major differences make the study of primordial nucleosynthesis interesting.

## 2.2 Why Primordial?

It is experimentally observed that the universe is mostly dominated by two elements namely Hydrogen($\tilde{7}5\%$) and Helium($\tilde{2}5\%$). This implies there is one neutron for every 7 protons (i.e. $\frac{n}{p} \approx \frac{1}{7}$). We need theoretical explanation to this ratio. One might think that this $He$ could have been produced in the stars through the chain of nuclear reactions. We can roughly calculate the order of $He$ produced in stars.

## 2.3 Helium Production in Stars

Let us assume that a $He$ nucleus is created by fusing 4 protons in the stars (though, one has to consider the whole network to calculate the exact rate of fusion)

$$4p \to {}^4He + 2e^+ + 2\nu_e$$

The mass difference for this reaction is $\sim 25.7MeV$, while the typical kinetic energy of neutrinos $\sim 0.4MeV$. Therefore about 25 MeV is released per four protons consumed. This is the energy that keeps the electron gas in our sun hot: energy is produced in the solar core at the required rate, just about balancing the energy that is carried off the sun by the photons emitted from the photosphere. Thus we can estimate the rate of fusion in the solar core from the measured solar constant.

solar constant $\sim 0.033cal/sec/cm^2$ distance to earth $\sim 1.49 \times 10^{13}cm = r$ therefore the power output is

$$(4\pi r^2)(.033cal/sec/cm^2) \sim 0.92 \times 10^{26}cal/sec \sim 2.4 \times 10^{39}MeV/sec$$

but as 4 protons are consumed for every 25 MeV produced implies $4 \times 10^{38}p/sec$ consumed

Mass of the sun is $\sim 1.19 \times 10^{57}protons$ The sun is roughly 5 billion years and burning at the current power level over that period. Then we can estimate the number of protons consumed over that lifetime

$$(3.15 \times 10^7 sec/year)(5 \times 10^9 years)(4 \times 10^{38}\ protons/sec) \sim 0.63 \times 10^{56}\ protons$$

But this is only $0.63/11.9 \sim 5.3\%$ of the sun's mass. Thus only $\sim 5\%$ of protons converted in 5 b.y. (this $He$ is also locked in the core of our sun, not in places like the inter-stellar medium where it could be counted by those interested in determining abundances.) And many protons are not in stars. Thus the tentative conclusion is that stellar burning contributes to, but cannot account for all, of the ${}^4He$. In fact, looking at the ${}^4He$ abundance as a function of stellar metallicity, stars with poor metallicity presumably were formed very early. The surfaces of such stars should not know about the ${}^4He$ synthesis in the core, but rather be representative of the star at its birth. So if the surface shows a large $^4He$ abundance it has to be primordial rather being formed in the stellar nucleosynthesis.

# 3 Thermal History of the Universe

## 3.1 Time Scale vs. Temperature

Gamow & Ralph Alpher in the 1940s proposed a big bang cosmology where the universe began as a hot soup, then expanded and cooled. When cooled below about $kT \sim 1MeV$, when $e^+, -e$ annihilation would occur, that soup would consists of the familiar stable particles like p, n, $e$ , and $\nu's$. The basic idea of big bang nucleosynthesis is a nuclear reaction network that begins with $n+p \rightarrow D+\gamma$ and this must happen within minutes as neutron's half-life is $(\tau_{1/2}(n)) \sim 10min$ So if there is no nucleosynthesis, there would be no neutrons now. That means neutrons exist in our present day world only because they bind in nuclei. This forms the most important idea in the theory of Big Bang Nucleosynthesis. Free neutrons have enough energy to decay to protons via beta decay. Bound neutrons do not because their binding energy makes this decay energetically impossible. So nuclei from the hot big bang must have freezed out which means the reaction rates have fallen below the expansion rate of the universe. To understand this further in terms of temperatures at which the dominant reactions and their respective time scales we can use the standard model of cosmology formalism of expanding universe. Recalling that in the early radiation dominated universe

$$\rho \sim a^{-4} \Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{4\dot{a}}{a} = -4\left(\frac{8\pi G\rho}{3}\right)^{1/2}$$

$$\Rightarrow t = \left(\frac{3}{32\pi G\rho}\right)^{1/2}$$

$$\Rightarrow t = \left(\frac{c^2}{48\pi GaT^4}\right)^{1/2}$$

$$= 1.09secs. \left[\frac{T}{10^{10}K}\right]^{-2}$$

Which gives us the following correlation between temperature and time.

| | |
|---|---|
| Temperature | — Time |
| $10^{11}K$ | — 0.01secs |
| $10^{10}K$ | — 1.07secs |
| $10^{9}K$ | — 3 mins |
| $10^{8}$ | — 5.3 hrs |
| $4 \times 10^{3}K$ | — $10^{5}$ yrs |

### 3.2 Thermal History of Early Universe

Assume that the early universe is hot & consider particles in thermal equilibrium at that temperature. It so happened that the particles outnumbered anti-particles causing matter-anti-matter asymmetry at the beginning. Hence, only the particle distributions are taken into account in calculating nuclear reactions.

$T > 10^{12}K$: The soup consists of $\gamma$,leptons,mesons,n,p,$\bar{n}$,$\bar{p}$ this era is difficult to study because of the strong interactions of the quark gluon plasma. It should be noted that the inflation time scale is of the order of $10^{-33}s$ and is before the process of baryogenesis.

$T \sim 10^{12}K$: constitution is $\gamma$,$\mu^+$,$\mu^-$,$e^\pm$,$\nu$, $\bar{\nu}$'s and small contamination of $n, p$ & $N_n \approx N_p$

$T < 10^{12}K$: $\mu^+$,$\mu^-$ annihilation happens. All $\mu$'s dissipate at $T \sim 10^{11}K$; $\nu, \bar{\nu}$'s decouple from leptons.

Below $10^{11}K$ mass difference of $n, p \Rightarrow$ more *protons* than neutrons.

$\sim 5 \times 10^9 K$ ($t \sim 4$ *sec*) $e^+$,$e^-$ annihilate and heat up photons. $\dfrac{n}{p} \approx \dfrac{1}{5}$.

At around $\sim 10^9 K$ $n$'s & $p$'s combine together to give nucleii.

At $\sim 4000K$ electrons captured by nucleii.

The photons there after freely stream and are observed in the microwave frequencies, forming the isotropic Cosmic Microwave Background.

## 3.3 Weak Interactions

Consider the equilibrium condition at high temperatures. If at some temperature there are particles in thermal equilibrium. no. density of $i^{th}$ species of particles with momentum between $q$ & $q + dq$ is

$$n_i(q) = \frac{g_i q^2 dq}{h^3} 4\pi \left[ \frac{1}{exp\left(\frac{E_i(q)-\mu_i}{kT}\right) \pm 1} \right]$$

$+$ sign for fermions & $-$ for bosons. $E_i(q) = (m_i^2 + q^2)^{1/2}$

$\mu_i$ = chemical potential, it is additivity & is conserved in all reactions

Therefore $\mu_\gamma = 0$ & $\mu_{particle} = -\mu_{anti\ particle}$ From

$$e^- + \mu^+ \rightarrow \nu_e + \nu_\mu$$

$$e^- + p \rightarrow \nu_e + n$$

$$\mu^- + p \rightarrow \nu_\mu + n$$

we can write

$$\mu_e - \mu_{\nu_e} = \mu_{\mu^-} - \mu_{\nu_\mu} = \mu_n - \mu_p$$

Thus there are 4 conserved intrinsic quantum numbers: charge, baryon number, $l_e$ (no. of $e^-$&$\nu_e$ minus $e^+$&$\bar{\nu_e}$), $l_\mu$ (no. of$\mu^-$&$\nu_\mu$ minus $\mu^+$&$\bar{\nu_\mu}$). Thus we have 4 independent chemical potentials. There are 4 chemical potentials taken as $\mu_p$,$\mu_e$,$\mu_{\nu_e}$,$\mu_{\nu_\mu}$ are determined by charge density, baryon number density, electron & muon number density. All $\sim a^{-3}$; $n_B << n_\gamma$.

Though $n_\nu$ is not known it is a good first approximation to take all 4 conserved quantities as $\approx 0$

For relativistic particles $E = q$, $p = \frac{1}{3}\rho$, we know that

$$\rho \propto T^4$$

$$T \propto \frac{1}{a}$$

Consider now an epoch when $kT \leq m_\pi$ i.e. $T < 1.5 \times 10^{12} K$. The particle distribution functions for electron & muon are given by

$$n_e(q)dq = n_e(q)dq = \frac{8\pi}{h^3} q^2 dq \left[ exp\left( \frac{\sqrt{q^2 + m_e^2}}{kT} \right) + 1 \right]^{-1}$$

$$n_\mu(q)dq = n_\mu(q)dq = \frac{8\pi}{h^3} q^2 dq \left[ exp\left( \frac{\sqrt{q^2 + m_\mu^2}}{kT} \right) + 1 \right]^{-1}$$

$\nu$'s are produced/destroyed/scattered in the following reactions

$$e^- + \mu^+ \leftrightarrow \nu_e + \bar{\nu}_\mu;\ e^+ + \mu^- \leftrightarrow \bar{\nu}_e + \nu_\mu$$

$$\nu_e + \mu^- \leftrightarrow \nu_\mu + e^-;\ \bar{\nu}_e + \mu^+ \leftrightarrow \bar{\nu}_\mu + e^+$$

$$\nu_\mu + \mu^+ \leftrightarrow \nu_e + e^+;\ \bar{\nu}_\mu + \mu^- \leftrightarrow \bar{\nu}_e + e^-$$

For $kT < m_\mu$:

Cross-section for all the above reactions is $\sigma_{\omega k} \approx \frac{g_{wk}^2}{\hbar^4}(kT)^2$, where

$$g_{wk} = 1.4 \times 10^{-49} erg - cm^3$$

is weak coupling constant.

All particles are of a speed roughly close to the velocity of light $c$

$$\Rightarrow n_{e^\pm,\mu^\pm} \approx \left( \frac{kT}{\hbar} \right)^3$$

Therefore rate of single $\nu$ scattering & the rate of $\nu$ production per charged lepton is of the order of

$$\sigma_{wk} n_l \approx g_{wk}^2 \hbar^{-7} (kT)^5$$

Total energy density $\rho \approx kT \left( \frac{kT}{\hbar} \right)^3$.

Expansion rate $H = \frac{\dot{a}}{a} = \sqrt{G\rho} \approx G^{-1/2} \hbar^{-3/2} (kT)^2$

when $kT > m_\mu$

On the other hand, when $kT < m_\mu$

$$\frac{\sigma n_l}{H} \approx G^{-1/2}\hbar^{-11}2c^{-7/2}g_{wk}^2(kT)^3 \approx \left(\frac{T}{10^{10}K}\right)^3$$

no. density of particles with $E > m_\mu$ is reduced by $exp(-m_\mu/kT)$

$$\frac{\sigma n_l}{H} \approx \left(\frac{T}{10^{10}K}\right)^3 exp\left(-\frac{10^{12}K}{T}\right)$$

Therefore, all reactions involving $\mu$'s decouple at $T \approx 1.3 \times 10^{11}K$

All reactions not involving $\mu$'s decouple at $T \approx 10^{10}K$.

The $\nu$-reactions decouple $\nu$'s are still relativistic gas & therefore its effective temperature $T_\nu$ keeps falling as does the photon temperatures. Thus $aT_\gamma = aT = aT_\nu = const.$.

Between $10^{12}K > T > 5 \times 10^9 K$, the gas consists of $\gamma, e^\pm, \nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ all relativistic particles (can add $\nu_\tau, \bar{\nu}_\tau$)

$$\Rightarrow \rho_{\nu_e} = \rho_{\bar{\nu}_\mu} = \rho_{\nu_\mu} = \rho_{\bar{\nu}_\mu} = \rho_\nu$$

$$\rho_\nu = \frac{4\pi}{h^3}\int_0^\infty q^3 dq \left[exp\left(\frac{q}{kT}\right)+1\right]^{-1} = \frac{7\pi^5}{30h^3}(kT)^4 \equiv \frac{7}{16}\alpha T^4$$

Where $\alpha = \dfrac{8\pi^5 k^4}{15h^3c^3} = 7.56 \times 10^{-15} erg\ cm^{-3}K^{-4}$

$$\rho_\gamma = \alpha T^4$$

For $kT > m_e$, $e^\pm$ are relativistic

$$\rho_{e^-} = \rho_{e^+} = 2\rho_\nu = \frac{7}{8}\alpha T^4$$

Therefore

$$\rho_{total} = \rho_{\nu_e} + \rho_{\bar{\nu}_e} + \rho_{\nu_\mu} + \rho_{\bar{\nu}_\mu} + \rho_{e^-} + \rho_{e^+} + \rho_\gamma = \frac{9}{2}\alpha T^4$$

Can similarly write energy density for general temperatures exactly. General expression for $S$ is

$$S = \frac{a^3}{T}[\rho_{e^-} + \rho_{e^+} + p_{e^-} + p_{e^+} + \rho_\gamma + p_\gamma]$$

We know $\rho_\gamma + p_\gamma = \dfrac{4}{3}\alpha T^4$ and

$$\rho_{e^-} = \int E(q)n(q,T)dq$$

and

$$p_{e^-} = \frac{q^2}{3E(q)} n(q,T) dq$$

where $E = \sqrt{q^2 + m^2}$ and

$$n(q,T) = \frac{8\pi}{h^3} \frac{q^2 dq}{\left[exp\left(\frac{\sqrt{q^2+m^2}}{kT}\right) + 1\right]}$$

$\frac{q}{kT} \to y$, $\frac{m}{kT} \to x$

$$\Rightarrow S = \frac{4}{3}\alpha(aT)^3 \xi(m/kT)$$

Where

$$\xi(m/kT) \equiv 1 + \frac{45}{(2\pi)^4} \int_0^\infty \frac{y^2 dy \left[\sqrt{x^2+y^2} + \frac{y^2}{3\sqrt{x^2+y^2}}\right]}{exp(\sqrt{x^2+y^2}) + 1}$$

$$\Rightarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} T \left[\xi\left(\frac{m_e}{kT}\right)\right]^{1/3}$$

It is easy to see that below $10^{10}K$ only $e^\pm$ & $\gamma$ in equilibrium with specific entropy in volume $a^3$:

$$s = \frac{a^3}{T}[\rho_{e^-} + \rho_{e^+} + p_{e^-} + p_{e^+} + \rho_\gamma + p_\gamma]$$

For $T > m_e$, $p_{e^\pm,\gamma} = \frac{1}{3}\rho_{e^\pm,\gamma}$, electrons & positrons being relativistic

$$\Rightarrow s = \frac{4}{3}\frac{a^3}{T}[\rho_{e^-} + \rho_{e^+} + \rho_\gamma] = \frac{11}{3}\alpha(aT)^3$$

Below $5 \times 10^9 K$, $e^+e^-$ annihilate, eventually leaving only photons in equilibrium.

$$s = \frac{4}{3}\frac{a^3}{T}\rho_\gamma = \frac{4}{3}\alpha(aT)^3$$

By conservation of entropy, one must have increase of $aT$ by a factor $\left(\frac{11}{4}\right)^{1/3} \approx (2.75)^{1/3}$

But $\nu$'s do not heat up as weak interactions are out of equilibrium and therefore $aT_\nu$ is unchanged. $\frac{aT_\nu}{aT}$ for $T < 10^9 K \to \left(\frac{4}{11}\right)^{1/3} \approx (1.401)^{-1}$

Therefore, $T_\gamma$ is 40% larger. So at present $T_\gamma \approx 2.7K \Rightarrow T_\nu \approx 1.9K$.

The density of gas gets contribution from $\gamma$'s and $\nu$'s so:

$\rho_R(\gamma + \nu' s) = \rho_{\nu_e} + \rho_{\bar{\nu}_e} + \rho_{\nu_\mu} + \rho_{\bar{\nu}_\mu} + \rho_\gamma$

$$= aT_\gamma^4 + \frac{7}{4}aT_\nu^4$$

$$= \left[1 + \frac{7}{4}\left(\frac{4}{11}\right)^{4/3}\right] aT_\gamma^4 \approx 1.45\alpha T_\gamma^4$$

Now the energy density of non-relativistic matter$= m_N n_N \sim a^{-3} \sim T_\gamma^3 = m_N n_N$

$$\Rightarrow n_N = n_{N_0}\left(\frac{T_\gamma}{T_{\gamma 0}}\right)^3$$

Therefore $m_N n_N = \rho_R$ at $T_C = \dfrac{m_N n_{N_0}}{1.45\alpha T_{\gamma_0}^3} = 4200K\left[\dfrac{m_N n_{N_0}}{10^{-30} g/cm^3}\right]$

Estimate of current matter density vary from $m_N n_{N_0} \approx 2 \times 10^{-29}$ to $3 \times 10^{-31} g/cm^3$

Therefore $T_C$ lies between 84,000K to 1200K

$T_{Recombination} \simeq 4000K$

# 4 Helium Synthesis

## 4.1 Neutron-proton abundance ratio

Nucleons weakly interact by the following reactions:

$$n + \nu_e \leftrightarrow p + e^-$$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

Recall the lepton number density expressions:

$$n_{e^\pm}(p)dp = \frac{8\pi}{h^3} p^2 dp \frac{1}{exp\left[\frac{E_e}{kT}\right] + 1}$$

$$n_{\nu,\bar{\nu}}(p)dp = \frac{4\pi}{h^3}p^2dp\frac{1}{exp\left[\frac{E_\nu}{kT}\right]+1}$$

Where $E_e = \sqrt{p^2+m_e^2}$ & $E_\nu = p$

Rates of reactions are given by the V-A theory. Pauli's principle implies that the phase space availability is suppressed by the number of filled states.

$$1-\left[exp\left(\frac{E_e}{kT}\right)+1\right]^{-1} = \left[1+e^{-E_e/kT}\right]^{-1}$$

$$1-\left[exp\left(\frac{E_\nu}{kT}\right)+1\right]^{-1} = \left[1+e^{-E_\nu/kT}\right]^{-1}$$

The rates of the above weak interactions are evaluated in a standard manner.

Consider for example

$n+\nu \rightarrow p+e^-$

Rate of this process per nucleon is

$$\lambda(n+\nu\rightarrow p+e^-) = A\int v_eE_e^2\frac{p_\nu^2dp_\nu}{[e^{E_\nu/kT}+1]}\frac{\delta(E_e-E_\nu-Q)}{[e^{-E_e/kT}+1]}$$

with $A = \frac{d_{wk}^2}{2\pi^3\hbar^7}$ $\int p_e^2dp_e \rightarrow \int p_eEdE(p_e = v_eE)$ gives $\int v_eE_e^2$.

Overall conservation of Energy is taken care by $\int \delta[E_e-E_\nu-Q]$.

Adding up all the processes in which $n$ goes to $p$, and then $p$ goes to $n$, one finds numerically that all $p\rightarrow n$ reaction decouples at $T\approx 10^{10}K$

For $T > 10^{10}K$

$$\frac{\lambda(p\rightarrow n)}{\lambda(n\rightarrow p)} = exp\left(\frac{-Q}{kT}\right)$$

with $Q = m_n - m_p$

For equilibrium, the principles of detailed balance implies

$$\lambda(n\rightarrow p)\times neutron\ density = \lambda(p\rightarrow n)\times proton\ density$$

$$\frac{n_n}{n_p} = \frac{\lambda(p\rightarrow n)}{\lambda(n\rightarrow p)} = exp\left(\frac{-Q}{KT}\right)$$

$$X_n = \frac{n_n}{n_n+n_p} = \left[1+e^{Q/kT}\right]^{-1}$$

## 4.2 Nucleii Abundances

$$n_i = \int n_i(q)dq = \frac{4\pi g_i}{h^3}\int \frac{q^2dq}{exp\left(\frac{E_i(q)-\mu_i}{kT}\right)\pm 1}$$

For every non-relativistic nuclei, (very good approximation), the $\pm 1$ is ignorable

$$E_i(q) \approx m_i + \frac{q^2}{2m_i}$$

$$\Rightarrow n_i = \frac{4\pi g_i}{h^3} exp\left(\frac{\mu_i - m_i}{kT}\right)\int_0^\infty q^2dq exp\left(\frac{-q^2}{2m_ikT}\right)$$

$$= g_i\left(\frac{2\pi m_ikT}{h^2}\right)^{3/2} exp\left(\frac{\mu_i - m_i}{kT}\right)$$

Let there be a nucleus i of $Z_i$ p's & $(A_i - Z_i)$ n's in equilibrium

$$\Rightarrow \mu_i = Z_i\mu_p + (A_i - Z_i)\mu_n$$

$X_i = \frac{n_iA_i}{n_N}$; $X_n = \frac{n_n}{n_N}$; $X_p = \frac{n_p}{n_N}$
Where $n_N$= total no. density of nucleons (bound or free)

$$n_N = n_{N_0}\left(\frac{a_0}{a}\right)^3 = \frac{\rho_{N_0}}{m_N}\left(\frac{a_0}{a}\right)^3$$

The expressions for $n_p, n_n, n_i$ are

$$n_p = 2\left(\frac{2\pi m_pkT}{h^2}\right)^{3/2} exp\left[\frac{\mu_p - m_p}{kT}\right]$$

$$n_n = 2\left(\frac{2\pi m_nkT}{h^2}\right)^{3/2} exp\left[\frac{\mu_n - m_n}{kT}\right]$$

$$n_i = g_A\left(\frac{2\pi m_AkT}{h^2}\right)^{3/2} exp\left[\frac{\mu_i - m_i}{kT}\right]$$

From

$$exp\left(\mu_i/kT\right) = exp\left[(Z_im_p + (A_i - Z_i)m_n)/kT\right]$$

$$\approx n_p^Z n_n^{A-Z}\left(\frac{2\pi}{m_NkT}\right)^{3A/2} 2^{-A} exp\left[(Z_im_p + (A_i - Z_i)m_n)/kT\right]$$

but $B_i = Z_i m_p + (A_i - Z_i)m_n - m_A$

$$\Rightarrow n_{A_i} = g_A A^{3/2} 2^{-A} \left(\frac{2\pi}{m_N kT}\right)^{3(A-1)/2} n_p^{Z_i} n_n^{A_i - Z} exp(B_A/T)$$

For closure density $\rho_c = \dfrac{3H^2}{8\pi G}$ for $H = h \times 100km/sec/Mpc$ & $\rho_B = \Omega_B \rho_c$

$$\eta = 2.68 \times 10^{-8} \Omega_B h^2$$

$n_B = \dfrac{\Omega_B \rho_c}{m_N}$ and $n_\gamma = \dfrac{2\zeta(3)}{\pi^3}\left(\dfrac{kT}{c\hbar}\right)^3$

$$X_i = \frac{n_{A_i}}{n_N} = g_A A^{3/2} 2^{-A} \left(\frac{2\pi}{m_N kT}\right)^{3(A-1)/2} X_p^{Z_i} X_n^{A_i - Z_i} (n_N)^{A_i - 1} exp[B_A/T]$$

Next use expression for $n_\gamma$ to get

$$X_i = g_i \left[\zeta(3)^{A-1} \Pi^{(1-A)/2} 2^{(3A-5)/2}\right] A^{5/2} \left(\frac{kT}{m_n}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} exp\left[B_i/kT\right]$$

where $\eta = \dfrac{n_N}{n_\gamma}$
$g_i = 2,\ A = 2,\ B_D = 2.2MeV$

For Deuterium,

$$X_D = g_i O(1) 2^{(3A-5)/2} A^{5/2}$$

$$\Rightarrow X_D \simeq 16 \left[\frac{kT}{m_n}\right]^{3/2} \eta exp\left[\frac{B_D}{T}\right] X_n X_p$$

For $\Rightarrow X_D \approx 1$ for $X_n$,$X_p \sim O(1)$

$$\Rightarrow 0 \approx \frac{3}{2}(A-1)\ln\left[\frac{kT}{m_N}\right] + (A-1)\ln\eta + \frac{B_D}{T}$$

$$\Rightarrow \frac{B_D}{T(A-1)} = \ln(\eta^{-1}) + 1.5\ln\left[\frac{m_N}{kT}\right]$$

$$\Rightarrow T = \frac{B/(A-1)}{\ln(\eta^{-1}) + 1.5\ln(m_N/kT)}$$

For $\eta \sim 10^{-9}$ we get $T_D \sim 0.07 MeV$

At this temperature, $X_D \approx 1$. However, Deuterium never gets to such high values as it gets involved in a nuclear reaction network. Binding Energy of ${}^2H \simeq 2.2 MeV$

$$p + n \leftrightarrow {}^2H + \gamma$$

$${}^2H + n \rightarrow {}^3H + \gamma$$

$${}^3H + p \rightarrow {}^4He + \gamma$$

$${}^2H + p \rightarrow {}^3He + \gamma$$

$${}^3He + n \rightarrow {}^4He + \gamma$$

$${}^3He + {}^2H \rightarrow {}^4He + p$$

etc The first reaction in the above chain is reversible till temperature is below $10^9 K$ after which there exits too few $\gamma$ with large enough energy to dissipate ${}^2H$. Subsequently remaining neutrons land up as ${}^4He$ once the Deuterium starts getting consumed.

# 5 BBN Codes

Big bang nucleosynthesis process consists of a chain of more than 60 reactions considering 26 nucleides. The network of reactions is shown in Figure 1. This has to be simulated in a computer numerically to find out the nuclei abundances. The first and the most popular of the nucleosynthesis codes is the Kawano code(NUC123) written in Fortran 77 based on the paper by Wagnorr. It is extremely user-friendly with menu driven interface. The parameters for simulation as well as the physical parameters can be set or changed manually to run the code for different cases of interest. There are many other codes like AlterBBN (written in C) however, the underlying algorithm remains the same.

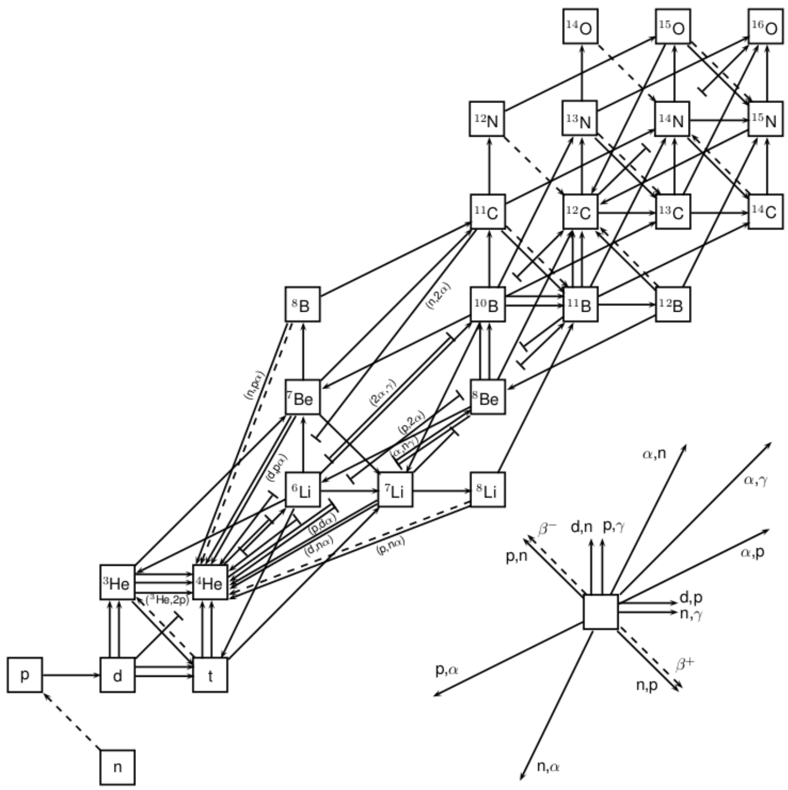


Figure 1: Network of Reactions simulated in NUC123

## 5.1 Reaction Rates

The total rate of change of abundance of nucleus i is given by the following first-order differential equation

$$\frac{1}{A_i}\frac{dX_i}{dt} = \pm \sum_j \frac{X_j}{A_j}\lambda_k(j) \pm \sum_{jk} \frac{X_j}{A_j}\frac{X_k}{A_k}[jk] \pm \sum_{jkl} \frac{X_j}{A_j}\frac{X_k}{A_k}\frac{X_l}{A_l}[jkl]$$

Here $\lambda_k$ is the reaction rate of the kth species. Similarly [jk] is the rate of reaction between jth and kth species calculated from their reactions thermally averaged cross-section that takes care of the velocity distribution of the interacting nuclei at a given temperature. For all the reactions shown in the Figure there are these coupled differential equations to be solved. The whole network of reactions is solved by two-step Runge-Kutta method with evolving time/temperature parameter.

## 5.2 Resonant vs. Non-Resonant

Nuclear reaction rates are calculated for both resonant and non-resonant reactions. It is very important to consider the resonant reaction rates as their cross-section values tend to be higher by the orders of magnitude in comparison to non-resonant reactions. They are important when the energy is higher than the most effective energy for thermonuclear reactions. Hence, relevant nuclear models are taken into account in the nucleosynthesis codes to account for both these types of reactions. very important to consider the resonant reaction rates as their cross-section values tend to be higher by the orders of magnitude in comparison to non-resonant reactions. They are important when the energy is higher than the most effective energy for thermonuclear reactions. Hence, relevant nuclear models are taken into account in the nucleosynthesis codes to account for both these types of reactions.

## 5.3 Simulation Parameters

As NUC123 is menu-driven application, one can set the parameters in the code by choosing relevent options in the menu. There are Computational parameters as well as the model parameters. Both types of parameters are briefly explained below.

### 5.3.1 Computational Parameters

As the coupled differential equations in Kawano code are solved using Runge-Kutta method, one can change the initial time step, time-step limiting constant etc. by choosing the concerned option in the menu to solve the equations with desired accuracy. We can also choose to set the initial and final temperatures of the simulation as multiples of $10^9$ K. The default run is set to $10^{11}$ K to $10^7$ K. To avoid singularities in the coupled differential equations matrix, we need to take a non-zero initial abundances of the nucleii and it also serves as the minimum amount of abundance below which the nucleii can be ignored. non-zero initial abundances of the nucleii and it also serves as the minimum amount of abundance below which the nucleii can be ignored.

### 5.3.2 Model Parameters

In the model parameters submenu, one can play God by choosing various parameters such as gravitational constant, neutron lifetime, no. of neutrino species, final Baryon-to-photon ratio, cosmological constant and neutrino degeneracies. The default values are set to the known standard values. Most of these parameters need not be changed. However, if one wants to check the nucleosynthesis code for alternative models, one has to make modifications in the relevant parts of the code. One can also vary the model parameters linearly and do multiple runs.

# 6 Results & Conclusion

The Nucleii abundances calculated through NUC123 code are plotted in figure 2. One can notice the sudden spike in Deuterium production (shown in red) that lasts for few minutes causes raise in the other elemental abundances mainly Helium (shown in yellow).

The mass percentage or mass fractions of Hydrogen (in green) & Helium can be seen in figure 3. As it is clearly seen the neutron percentage reduction (shown in blue) causes spike in the Helium mass fraction(yellow colored).

Hence, in this project, the algorithm of Big Bang Nucleosynthesis synthesis codes is studied. Simulations varying different parameters is done a few of which is presented in the report. The nucleii abundances in the universe is systematically understood with the help of the results.

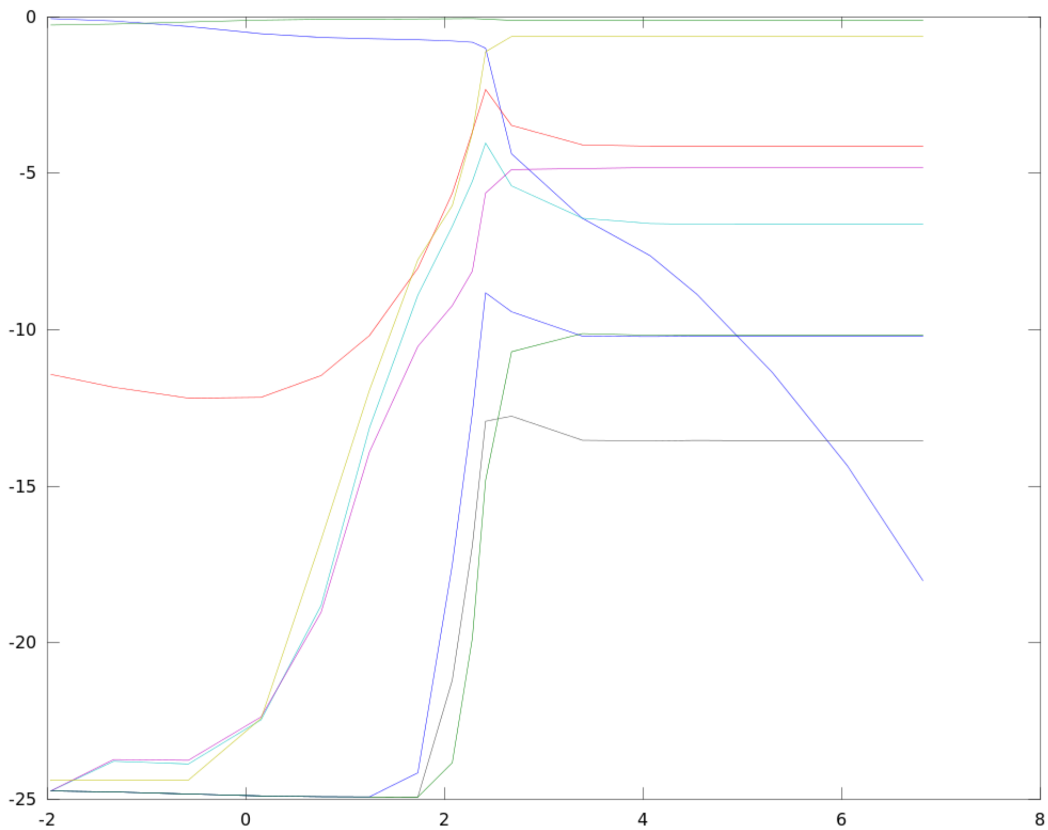


Figure 2: Nucleii Abundances vs. Time (Log)

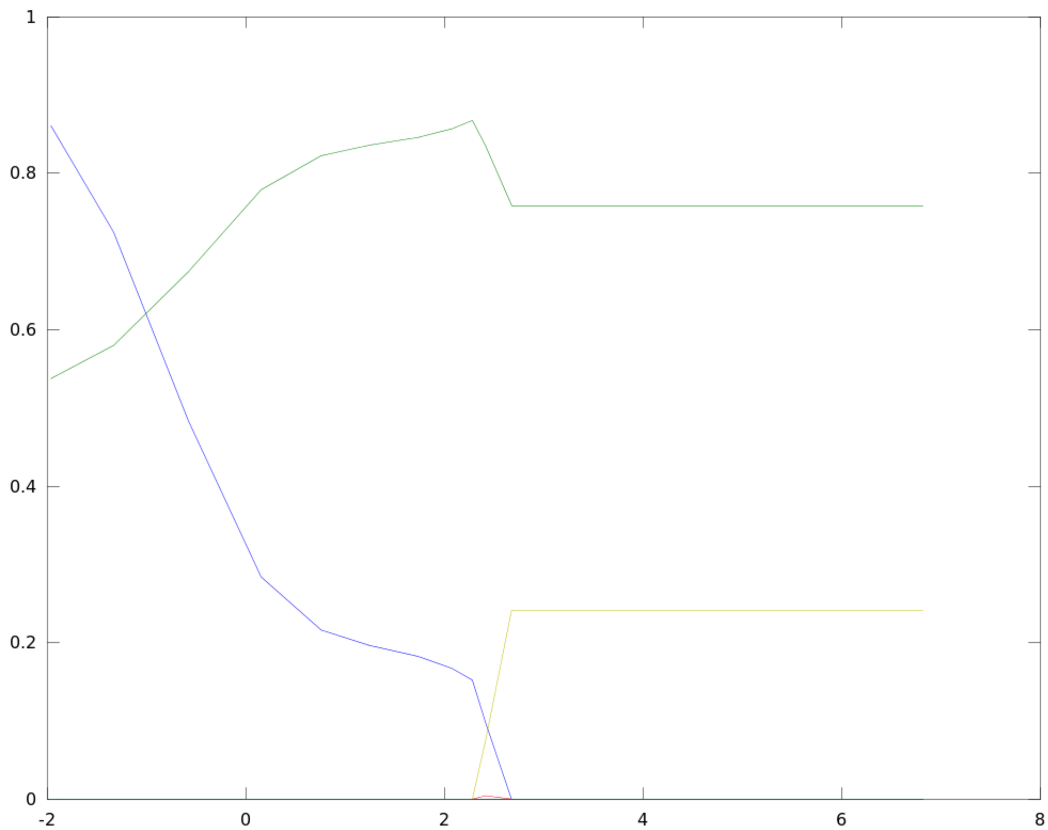


Figure 3: Helium, Hydrogen & Neutron mass fractions vs. Time (Log)