A BRIEF INTRODUCTION TO

GRTensor

on MAPLE platform

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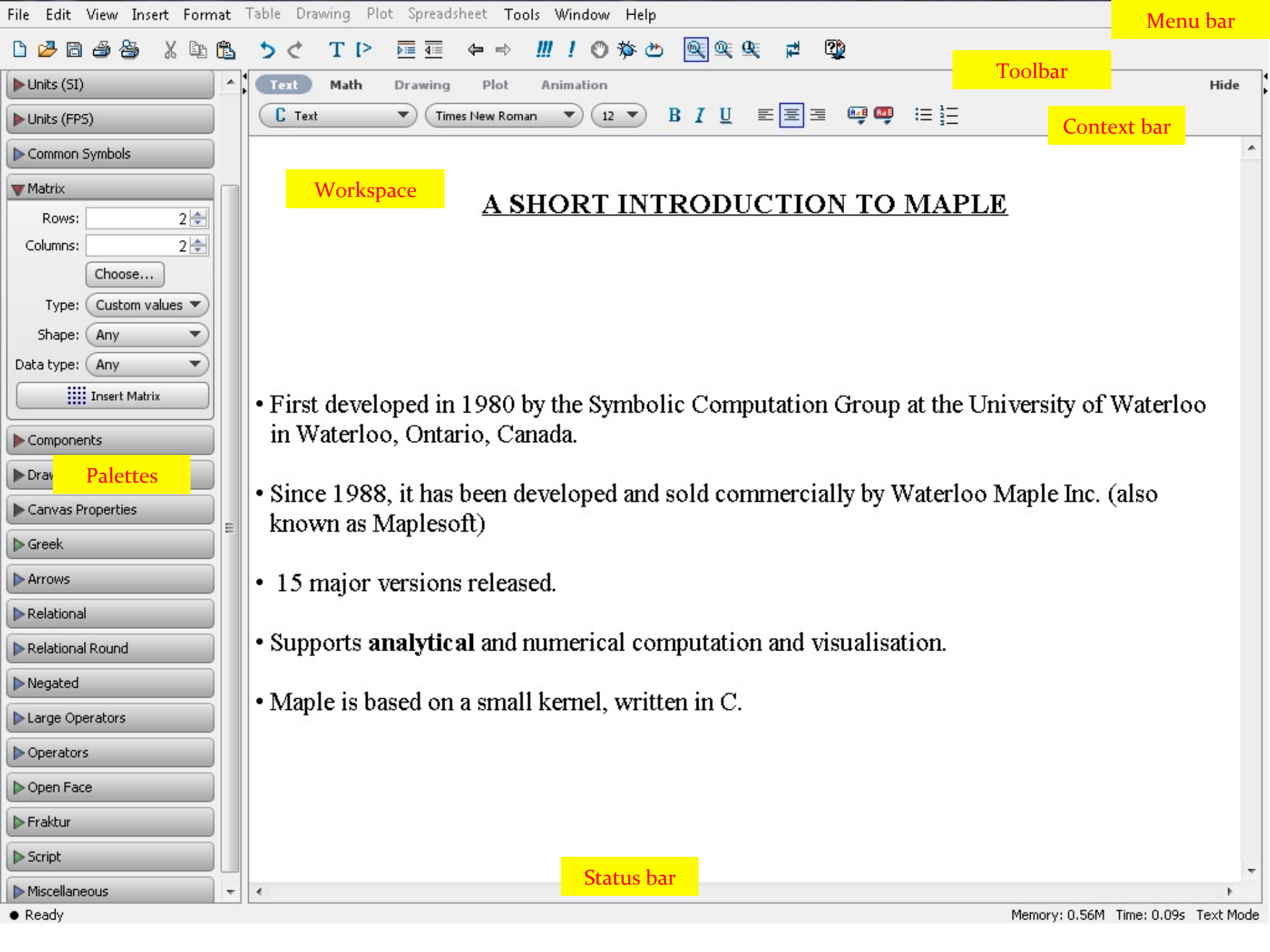
As a part of Ph.D. course PHYS 601

TOPICS ADDRESSED:

- HISTORY OF MAPLE
- INTRODUCTION TO INTERFACE
- OPERATIONS POSSIBLE
- BENEFITS/DRAWBACKS
- TENSORS
- INTRODUCTION TO GRTensor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A SHORT INTRODUCTION TO MAPLE
• First developed in 1980 by the Symbolic Computation Group at the University of Waterloo in Waterloo, Ontario, Canada.
• Since 1988, it has been developed and sold commercially by Waterloo Maple Inc. (also known as Maplesoft)
• 15 major versions released.
• Supports analytical and numerical computation and visualisation.
• Maple is based on a small kernel, written in C.
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Memory: 0.56M Time: 0.09s Text Mode

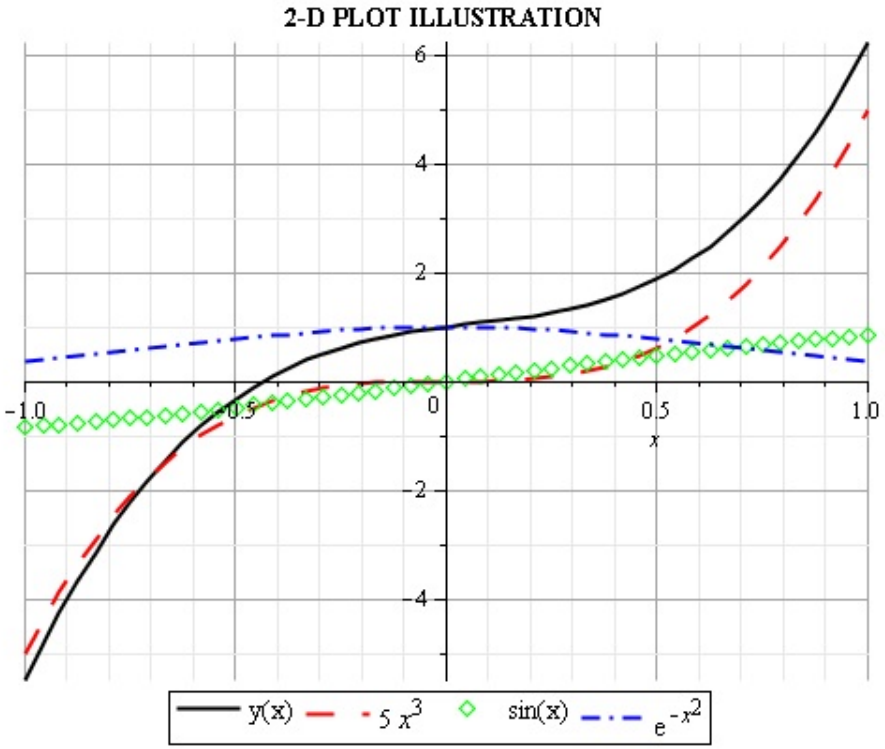
$y := 5x^3 + \sin(x) + \exp(-x^2);$

$$5x^3 + \sin(x) + e^{-x^2} \tag{1}$$

$Int(y, x) = int(y, x);$

$$\int \left(5x^3 + \sin(x) + e^{-x^2}\right) dx = \frac{5}{4}x^4 - \cos(x) + \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(x) \tag{2}$$

$plot([y, 5x^3, \sin(x), \exp(-x^2)], x = -1..1);$



$$ode := diff(y(x), x, x) = 2 * y(x) + 1;$$

$$\frac{d^2}{dx^2} y(x) = 2\,y(x) + 1 \quad (1)$$

← O.D.E.

$$dsolve(ode);$$

$$y(x) = e^{\sqrt{2}\,x}\,\_C2 + e^{-\sqrt{2}\,x}\,\_C1 - \frac{1}{2} \quad (2)$$

← Analytic soln.

$$ics := y(0) = 1, D(y)(0) = 0;$$

$$y(0) = 1, D(y)(0) = 0 \quad (3)$$

← Initial cond.

$$dsolve(\{ode, ics\});$$

$$y(x) = \frac{3}{4} e^{\sqrt{2}\,x} + \frac{3}{4} e^{-\sqrt{2}\,x} - \frac{1}{2} \quad (4)$$

$$sol := dsolve(\{ode, ics\}, y(x), method = laplace);$$

$$y(x) = -\frac{1}{2} + \frac{3}{2}\cosh(\sqrt{2}\,x) \quad (5)$$

← Laplace mthd.

$$series\_sol := dsolve(\{ode, ics\}, y(x), series);$$

$$y(x) = 1 + \frac{3}{2}x^2 + \frac{1}{4}x^4 + O(x^6) \quad (6)$$

← Series soln.

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$f := \sin(x+y) - \exp(x) * y = 0;$

$g := x^2 - y = 2;$

$fsolve(\{f, g\});$

$$\sin(x+y) - e^x y = 0$$

$$x^2 - y = 2$$

$$\{x = -6.017327250,\ y = 34.20822723\} \quad (1)$$

$restart;$

$solve(\{a*x + b\cdot y = c,\ d\cdot x + e*y = f\}, [x, y]);$

Can work with undefined constants !!

$$\left[\left[x = -\frac{-e\,c + f\,b}{-d\,b + a\,e},\ y = \frac{-d\,c + a\,f}{-d\,b + a\,e}\right]\right] \quad (2)$$

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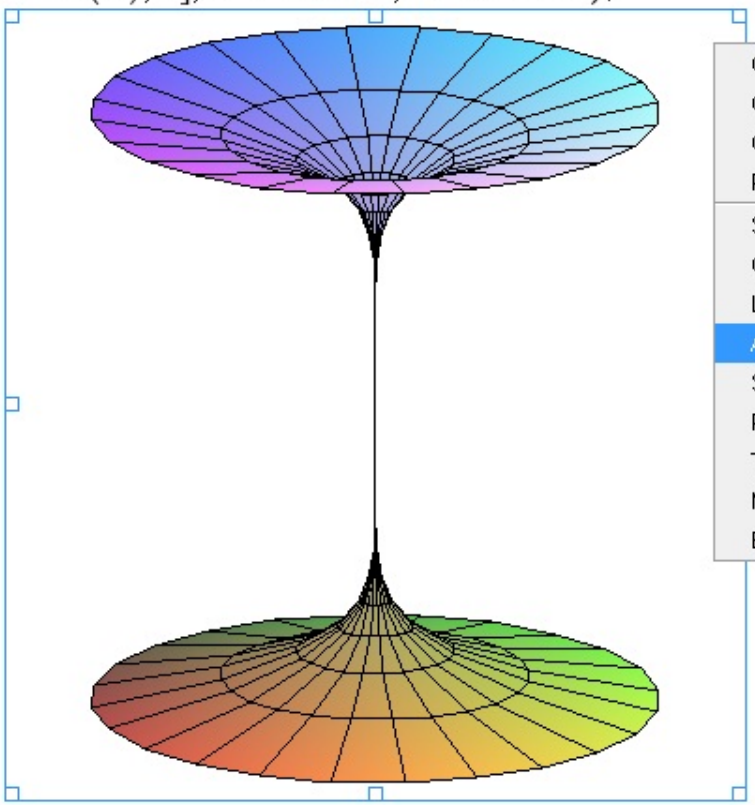
360° view

plot formatting options

$$plot3d([t^7 \cdot sin(u), t^7 \cdot cos(u), t], t = -20..20, u = 0..2\,Pi);$$



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Ready

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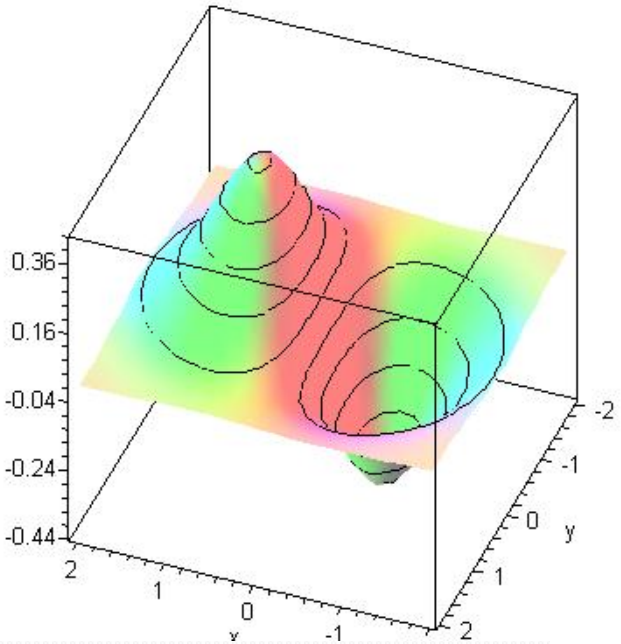
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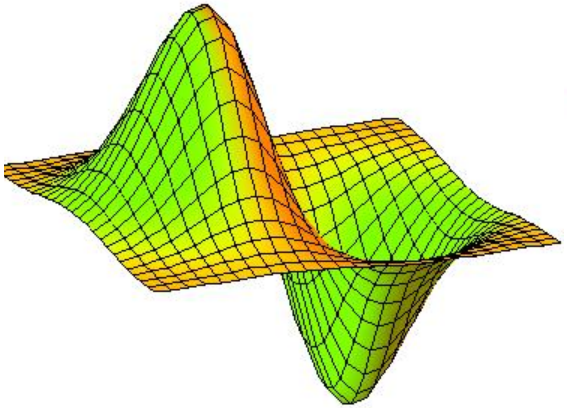

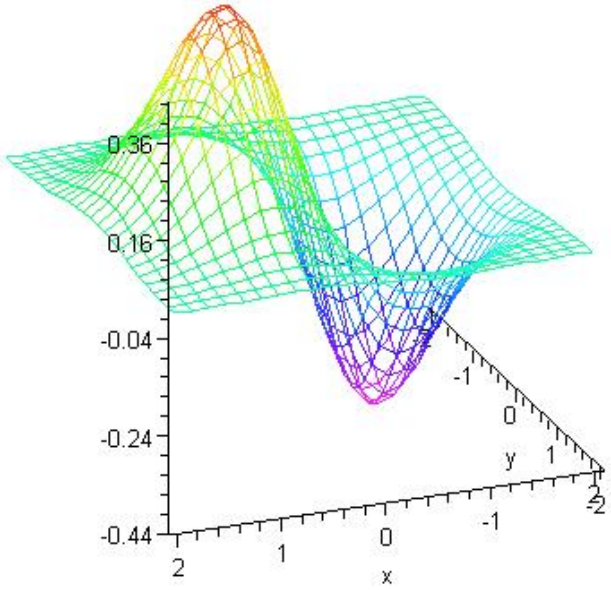
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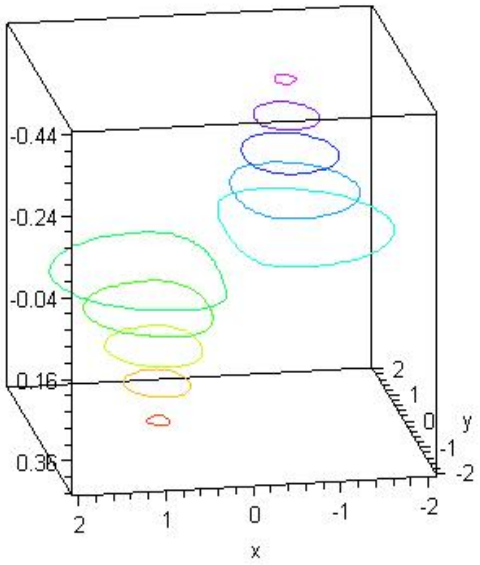
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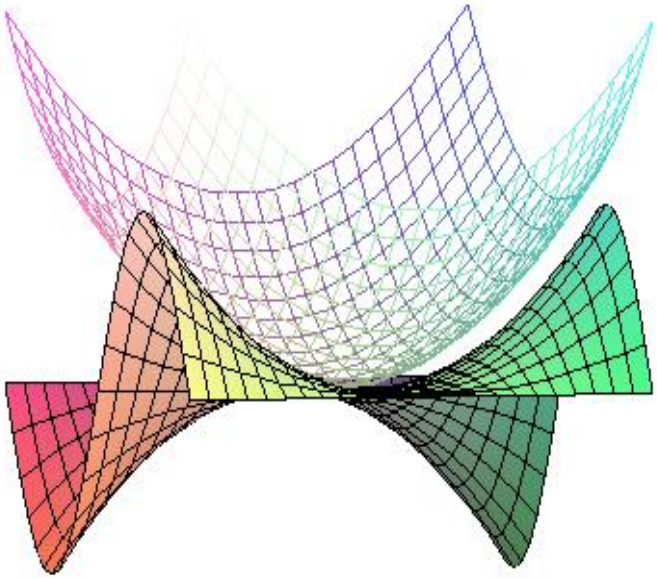

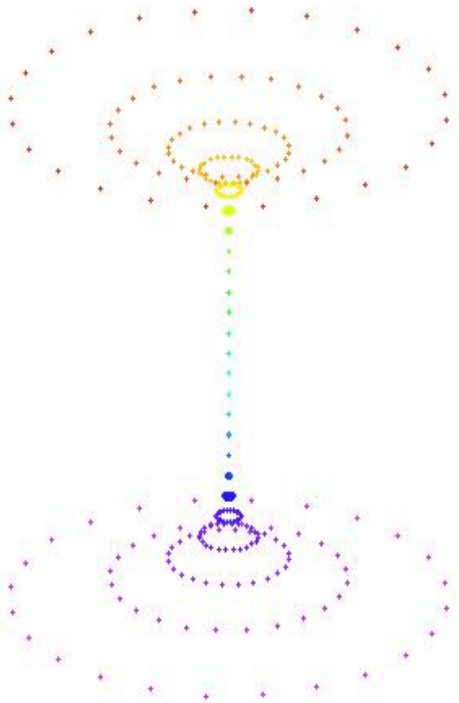

TENSORS

- An incomplete definition
- Tensors generally used in cosmology
- How are they obtained
- Need for a package like GRTensor

Line element (1)

Kerr Metric

$$ds^2 = \frac{\left(r^2 + a^2\cos(\theta)^2\right)\,dr^2}{r^2 - 2mr + a^2} + \left(r^2 + a^2\cos(\theta)^2\right)\,d\theta^2 + \sin(\theta)^2\left(r^2 + a^2 + \frac{2mra^2\sin(\theta)^2}{r^2 + a^2\cos(\theta)^2}\right)\,d\phi^2 - \frac{4mar\sin(\theta)^2\,d\phi\,dt}{r^2 + a^2\cos(\theta)^2} + \left(-1 + \frac{2mr}{r^2 + a^2\cos(\theta)^2}\right)\,dt^2 \quad (1)$$

$$R_{r\phi r\phi} = \left(\left(-14a^6\cos(\theta)^4 m r^4 + 11a^6 r^5\sin(\theta)^2\cos(\theta)^2 + 10a^6 r^5\cos(\theta)^4\sin(\theta)^2 - 2a^6 r^4\cos(\theta)^6 m - 7a^4 r^7\sin(\theta)^2 - 41a^6 m r^4\sin(\theta)^2\cos(\theta)^2 + 2a^8\cos(\theta)^2 r^3\sin(\theta)^4 - 6a^2 r^9\sin(\theta)^2 + a^8 r^3\cos(\theta)^8 + a^2 r^9 + 16a^6 r^4 m\sin(\theta)^4\cos(\theta)^2 + 7a^8\cos(\theta)^4 m r^2\sin(\theta)^4 + 3a^8 r^2\cos(\theta)^6 m\sin(\theta)^2 + 2r^{10}m - 3a^6 r^5\sin(\theta)^2 - r^{11} - 2a^2 r^8 m + 3a^{10} r\cos(\theta)^8 + a^{10} m\sin(\theta)^2\cos(\theta)^6 - a^{10}\cos(\theta)^4 m\sin(\theta)^2 + 7a^{10}\cos(\theta)^4 r\sin(\theta)^2 + 2a^{10}\cos(\theta)^4 r\sin(\theta)^4 + a^{10}\cos(\theta)^4 m\sin(\theta)^4 - 6a^8\cos(\theta)^6 m r^2 + 3a^8 r^3\cos(\theta)^6\sin(\theta)^2 + 23a^8 r^3\sin(\theta)^2\cos(\theta)^4 + 2a^8\cos(\theta)^4 r^3\sin(\theta)^4 + 4a^8\cos(\theta)^2 r^3\sin(\theta)^2 + 2a^6 r^5\cos(\theta)^6 + 10a^8\cos(\theta)^6 r^3 - 2a^2 r^9\cos(\theta)^2 + 6a^4\cos(\theta)^2 r^7 + 12a^6\cos(\theta)^4 r^5 + 24a^4 m^2 r^5\sin(\theta)^2\cos(\theta)^2 - 10a^6 r^4\cos(\theta)^4 m\sin(\theta)^2 - 15a^4 r^6\cos(\theta)^2 m\sin(\theta)^2 + 2a^6\cos(\theta)^2 r^5\sin(\theta)^4 - 5a^6 r^4 m\sin(\theta)^4 + 5a^6 m r^4\sin(\theta)^2 - 2a^4 r^6\cos(\theta)^4 m + a^4 r^7\cos(\theta)^2\sin(\theta)^2 - 10a^4\cos(\theta)^2 r^6 m + 5a^{10}\cos(\theta)^6 r\sin(\theta)^2 - 8a^4 m^2 r^5\sin(\theta)^2 + 13a^4 m r^6\sin(\theta)^2 + 8a^4 r^5 m^2\sin(\theta)^4 - 3a^4 r^6 m\sin(\theta)^4 + 2a^2 r^8 m\cos(\theta)^2 + 14a^2 r^8 m\sin(\theta)^2 - 8a^2 m^2 r^7\sin(\theta)^2 - 21a^8 m\sin(\theta)^2\cos(\theta)^4 r^2 - 12a^8\cos(\theta)^2 m r^2\sin(\theta)^2 + 8a^8\cos(\theta)^2 m r^2\sin(\theta)^4 + 4a^8\cos(\theta)^2 m r^2\sin(\theta)^6 + 24a^6\cos(\theta)^2 m^2\sin(\theta)^2 r^3 - 24a^6\cos(\theta)^2 m^2\sin(\theta)^4 r^3\right) m\sin(\theta)^2\Big) \Big/ \Big(\left(a^4\cos(\theta)^2 + r^2 a^2\cos(\theta)^2 - 2a^2 m r + a^2 r^2 + 2mra^2\sin(\theta)^2 + r^4 - 2r^3 m\right)\left(r^2 - 2mr + a^2\right)\left(r^2 + a^2\cos(\theta)^2\right)^3\Big) \quad (2)$$

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```
readlib(grii):
grtensor():
```

Initialization

GRTensorII Version 1.79 (R4)

6 February 2001

Developed by Peter Musgrave, Denis Pollney and Kayll Lake

Copyright 1994-2001 by the authors.

Latest version available from: http://grtensor.phy.queensu.ca/ (1)

```
qload(schw):
```

Loading a metric

```
Calculated ds for schw (0.000000 sec.)
```

Default spacetime = schw

For the schw spacetime:

Coordinates

$x(up)$

$$x^a = \left[r \;\; \theta \;\; \phi \;\; t \right]$$

Line element

$$ds^2 = \frac{dr^2}{1 - \frac{2m}{r}} + r^2\, d\theta^2 + r^2 \sin(\theta)^2\, d\phi^2 + \left(-1 + \frac{2m}{r}\right) dt^2$$

The Schwarzschild metric in curvature coordinates (2)

```
grcalc(Chr(up,dn,dn)):
Created definition for Chr(up,dn,dn)
Calculated detg for schw (0.000000 sec.)
```

Ready

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$grcalc(Chr(up, dn, dn))$:

```
Created definition for Chr(up,dn,dn)
Calculated detg for schw (0.000000 sec.)
Calculated g(up,up) for schw (0.016000 sec.)
Calculated g(dn,dn,pdn) for schw (0.000000 sec.)
Calculated Chr(dn,dn,dn) for schw (0.015000 sec.)
Calculated Chr(up,dn,dn) for schw (0.000000 sec.)
```

$CPU\ Time = 0.109$ (3)

Calculating christoffel's symbols

$grdisplay(Chr(up, dn, dn))$:

Display the result

For the schw spacetime:

Chr(up,dn,dn)

$$\Gamma^{r}{}_{r\,r} = -\frac{m}{(r-2\,m)\,r}$$

$$\Gamma^{\theta}{}_{\theta\,r} = -\frac{1}{r}$$

$$\Gamma^{\phi}{}_{\phi\,r} = -\frac{1}{r}$$

$$\Gamma^{t}{}_{t\,r} = -\frac{m}{(r-2\,m)\,r}$$

$$\Gamma^{\theta}{}_{r\,\theta} = \frac{1}{r}$$

$$\Gamma^{r}{}_{\theta\,\theta} = r-2\,m$$

$$\Gamma^{\phi}{}_{\ldots} = -\frac{\cos(\theta)}{\ldots}$$

Ready

Memory: 0.56M Time: 0.13s Math Mode

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Calculating Reimann tensor

```
grdisplay(R(dn, dn, dn, dn)):
```

For the schw spacetime:

Covariant Riemann

$$R_{r\theta r\theta} = -\frac{m}{r-2m}$$

$$R_{r\phi r\phi} = -\frac{m\sin(\theta)^2}{r-2m}$$

$$R_{rtrt} = -\frac{2m}{r^3}$$

$$R_{\theta\phi\theta\phi} = 2r\sin(\theta)^2 m$$

$$R_{\theta t\theta t} = \frac{(r-2m)m}{r^2}$$

$$R_{\phi t\phi t} = \frac{(r-2m)m\sin(\theta)^2}{r^2} \quad (6)$$

```
grcalc(R(dn, dn)):
Calculated Chr(dn,dn,up) for schw (0.016000 sec.)
Calculated R(dn,dn) for schw (0.016000 sec.)
```

CPU Time = 0.047 (7)

```
grdisplay(R(dn, dn)):
```

For the schw spacetime:

Covariant Ricci

$R(dn, dn)$

Ready

Memory: 0.56M Time: 0.13s Math Mode

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$grcalc(R(dn, dn))$:

```
Calculated Chr(dn,dn,up) for schw (0.016000 sec.)
Calculated R(dn,dn) for schw (0.016000 sec.)
```

$CPU\ Time = 0.047$ (7)

Ricci Tensor

$grdisplay(R(dn, dn))$:

For the schw spacetime:

Covariant Ricci

$R(dn, dn)$

$R_{a\,b} = All\ components\ are\ zero$ (8)

$grcalc(Ricciscalar)$:

```
Calculated Ricciscalar for schw (0.000000 sec.)
```

$CPU\ Time = 0.$ (9)

Ricci Scalar

$grdisplay(Ricciscalar)$:

For the schw spacetime:

Ricci scalar

$R = 0$ (10)

$grcalc(G(dn, dn))$:

```
Calculated Ricciscalar for schw (0.016000 sec.)
Calculated G(dn,dn) for schw (0.015000 sec.)
```

$CPU\ Time = 0.031$ (11)

$grdisplay(G(dn, dn))$:

Einstein Tensor

For the schw spacetime:

Covariant Einstein

$G(dn, dn)$

$G_{a\,b} = All\ components\ are\ zero$ (12)

Ready

Memory: 0.56M Time: 0.13s Math Mode

Default spacetime = newmetric

For the newmetric spacetime:

Coordinates

$x(up)$

$$x^{a} = \left[\, t \;\; r \;\; \theta \;\; \phi \,\right]$$

Line element

$$ds^2 = -dt^2 + dr^2 + \left(\left[r\right]^2 - c^2\left[t\right]^2\right) d\theta^2 + d\phi^2$$

The new metric

```
makeg() completed.
```

qload(*newmetric*) :

```
Default metric is now newmetric.
```

grcalc(*Chr*(*up*, *dn*, *dn*)) :

grdisplay(*Chr*(*up*, *dn*, *dn*)) :

For the newmetric spacetime:

Chr(up,dn,dn)

$$\Gamma^{\theta}{}_{\theta t} = -\frac{c^2\left[t\right]\left[1\right]}{-\left[r\right]^2 + c^2\left[t\right]^2}$$

$$\Gamma^{\theta}{}_{\theta r} = \frac{\left[r\right]\left[1\right]}{-\left[r\right]^2 + c^2\left[t\right]^2}$$

$$\Gamma^{\theta}{}_{t\theta} = \frac{c^2\left[t\right]\left[1\right]}{-\left[r\right]^2 + c^2\left[t\right]^2}$$

$$\Gamma^{\theta}{}_{r\theta} = -\frac{\left[r\right]\left[1\right]}{-\left[r\right]^2 + c^2\left[t\right]^2}$$

$$\Gamma^{t}{}_{\theta\theta} = c^2\left[t\right]\left[1\right]$$

$$\Gamma^{r} = \left[r\right]\left[1\right]$$

(2)

SYNTAX	RESULT
R(dn,dn,pdn)	$R_{ab,c}$
R(dn,d,cdn)	$R_{ab;c}$
> grdef (‘A{a b}’):	Creates a new vector ‘ A_{ab} ‘
> grcalc (A(dn,dn)):	Inputs the components of ‘ A_{ab} ‘
> grdef (‘A{^a ^b}’):	Creates a new vector ‘ A^{ab} ‘
> grdef (‘new object:= object definition’)	Defines a new tensor
R{^a ^b b c}	$\Sigma R^{ab}{}_{bc}$
R{^a ^b}*Box[R{ a b }]	$R^{ab}\,\square\, R_{ab}$

Some other jobs GRTensor can be used for :

- Defining new tensors
- Modifying tensor components
- Finding sum / products of tensors
- Tensor Calculus
- Simplifying the results
- Working in multiple geometries
- Many other operations Iam still unaware of…….