## A Brief Introduction to Mathematica

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## Outline:

-Running Mathematica
-Getting Started
-Working with the Notebook Interface
-Getting Information from Mathematica
-The Mathematica System

## The Structure of Mathematica

■ Mathematica $\rightarrow$ a modular software system

- The kernel which actually performs computations is separate from the front end which handles interaction with the user.
- To work on Mathematica is to use interactive documents known as notebooks.

■ Notebooks mix Mathematica input and output with text, graphics, palettes and other material.
■ Notebook interface
interactive documents
■ Text-based interface
text from the keyboard
■ MathLink interface communication with other programs
■ It can interact not only with human users but also with other programs $\rightarrow$ achieved primarily through MathLink $\rightarrow$ a standardized protocol for two-way communication between external programs and the Mathematica kernel.

## Screenshot of a blank notebook



## Using a Notebook Interface

■ A purely graphical interface $\rightarrow$ double-click the Mathematica icon to start Mathematica.

- A textually based operating system $\rightarrow$ type the command Mathematica to start.
- An empty notebook with a blinking cursor $\rightarrow$ by default will interpret your text as input $\rightarrow$ type Shift+Enter to make Mathematica process your input.
- Shift+Enter tells Mathematica that you have finished your input.

If numeric keypad $\rightarrow$ its Enter key instead of Shift+Enter.

- After you send Mathematica input from your notebook, Mathematica will label your input with $\ln [n]:=$. It labels the corresponding output Out[n]=. Labels are added automatically.
- $\ln [1]:=2+2$

$$
\text { Out[1]= } 4
$$

■ notebooks are part of the "front end" to Mathematica.

$$
\begin{aligned}
& 2+2 \\
& 4 \\
& 3-5 \\
& -2
\end{aligned}
$$

$$
\text { Solve }\left[x^{\wedge} 2-2 x+1==0, x\right]
$$

$$
\{\{\mathbf{x} \rightarrow 1\},\{\mathbf{x} \rightarrow 1\}\}
$$

Plot［Tan［x］，$\{x, 0, P i\}]$


## Getting Information about Mathematica Objects

■ ?Log
This gives information on the built-in function Log.
Log[z] gives the natural logarithm of $z$ (logarithm to base e).
Log[b,z] gives the logarithm to base b.

■ ?Name show information on Name ??Name show extra information on Name ?Aaaa* show information on all objects whose names begin with Aaaa

■ You can put * anywhere in the string you ask ? about. For example, ? *Expand would give you all objects whose names end with Expand. Similarly, ? $x^{*} 0$ would give you objects whose names start with x , end with 0 , and have any sequence of characters in between.
$\ln [2]:=? \log$
$\log [z]$ gives the natural logarithm of $z$ (logarithm to base 8 ).
$\log [b, z]$ gives the logarithm to base $b$.
$\ln [3]=$ ?? $\log$
$\log [z]$ gives the natural logarithm of $z$ (logarithm to base $s$ ),
$\log [b, z]$ gives the logarithm to base $b . \gg$
Attributes [Log] $=$ [Listable, NumericFunction, Protected
$\ln [4]=$ ? Export

Export["file,ext", expr] exports data to a file, converting it to the format corresponding to the file extension ext.
Export[ffle, expr, "format"] exports data in the specified format.
Export[file, exprs, elems] exports data by treating exprs as elements specified by elems. $\Rightarrow$
$\ln [5]=$ ? ? Export

Export[ f file,ext", expr] exports data to a file, converting it to the format corresponding to the file extension ext.
Export[ffle, expr, "format"] exports data in the specified format.
Export[file, exprs, elems] exports data by treating exprs as elements specified by elems. *

Attributes [Export] $=\{$ Protected, ReadProtected $\}$
$4 i>$ 分 $F[\ldots]$ SEARCH Export

Q Search for all pages containing Export.
MORE INFORMATION
^ EXAMPLES
^ Basic Examples (3)
Export an image object:

In[1]]:= Export["landscape.jpg",


Out[1]= landscape.jpg

## Export a plot as a GIF:

$\operatorname{In}[1]:=\operatorname{Export}[$ "test.gif", $\operatorname{Plot}[\operatorname{Sin}[x],\{x, 0,10\}]]$
Out[1]= test.gif

Export a formula as a GIF:
In[1]:= Export["test.gif", Integrate [1/( $\left.\left.\mathrm{x}^{\wedge} 4-1\right), \mathrm{x}\right]$ ]
Out[1]= test.gif

## Warnings and Messages

- If it looks as if Mathematica is doing something you definitely did not intend, Mathematica will usually print a message to warn you.
e.g.the square root function should have only one argument. Mathematica prints a message to warn you that you have given two arguments here.
- $\operatorname{In}[1]:=$ Sqrt[4, 5]

During evaluation of $\operatorname{In}[1]:=$ Sqrt::argx: Sqrt called with 2 arguments; 1 argument is expected. >>
Out[1]= Sqrt[4, 5]
■ You can suppress the message for one evaluation using Quiet. $\operatorname{In}[2]:=$ Quiet[Sqrt[4, 5]]
Out[2]= Sqrt[4, 5]

## Interrupting Calculations

■ There will probably be times when you want to stop Mathematica in the middle of a calculation. Perhaps you realize that you asked Mathematica to do the wrong thing. Or perhaps the calculation is just taking a long time, and you want to find out what is going on.

■ The way that you interrupt a Mathematica calculation depends on what kind of interface you are using.

■ Alt+,

- Ctrl+C
notebook interfaces
text-based interfaces


## The Mathematica System

■ Getting Used to Mathematica
■ Differences between Computer Systems
■ The Limits of Mathematica

## Getting Used to Mathematica

## $\square$ Some Important Rules:

Arguments of functions are given in square brackets.

- Names of built-in functions have their first letters capitalized.
- Multiplication can be represented by a space.
- Powers are denoted by ${ }^{\wedge}$.
- Numbers in scientific notation are entered, for example, as 0.00025 or $2.5^{* 1} 10^{\wedge}-4$.

Some examples: $\operatorname{Sin}[x]$
Each of these represents multiplication.

$$
\text { a*b ab a(b+1); } 2 x \text { means } 2^{*} x
$$

- Uppercase and lowercase letters are recognized as different characters. Lists are wrapped with curly brackets.
- Commas are used to separate arguments.


## Differences between Computer Systems

■ Elements of Mathematica that are exactly the same on all computer systems:
The language used by the Mathematica kernel
The structure of Mathematica notebooks
The MathLink communication protocol

- Elements that can differ from one computer system to another:

The visual appearance of windows, fonts, etc.
Mechanisms for importing and exporting material from notebooks Keyboard shortcuts for menu commands
■ Communications between the kernel and the front end are handled by MathLink.

## The Limits of Mathematica

- In just one Mathematica command, you can easily specify a calculation that is far too complicated for any computer to do. For example, you could ask for $\operatorname{Expand}\left[(1+x)^{\wedge}\left(10^{\wedge} 100\right)\right]$. The result of this calculation would have $10^{\wedge} 100+1$ terms--more than the total number of particles in the universe.
- If your computer does run out of memory in the middle of a calculation, most versions of Mathematica have no choice but to stop immediately. As a result, it is important to plan your calculations so that they never need more memory than your computer has.
$\square$ Memory space is the most common limiting factor in Mathematica calculations. Time can also, however, be a limiting factor. You will usually be prepared to wait a second, or even a minute, for the result of a calculation. But you will less often be prepared to wait an hour or a day, and you will almost never be able to wait a year.


## Core Language "How to" Topics

■ Create Definitions for Variables and Functions »
■ Clear My Definitions »
■ Map a Function over a List »
■ Work with Pure Functions »
■ Enter Ranges and Options for Functions »

## How to Create Definitions for Variables and Functions

- Here is a simple transformation rule. It says: whenever you see x , replace it by 3:
$\ln [31]:=x=3$
Out[31]= 3
- The variable x has a value of 3 .
$\ln [32]:=x^{\wedge} 2$
Out[32]= 9
■ Functions in Mathematica are defined by rules that act on patterns.

$$
f[x]:=x^{\wedge} 2
$$

■ The rule says: if you have f of any expression, replace it by that expression squared:
$\ln [46]:=$ f[expr]
Out[46]= expr ${ }^{2}$

- $g[x, y]:=f[x]+f[y]$
$\ln [49]:=\mathrm{g}[3,4]$
Out[49]= 25

■ Always use := to define functions, otherwise the variables on the righthand side may not represent the associated expressions on the left-hand side, since they will be evaluated before the rule is defined.

```
Untitled-1 *
```

    \(\ln [6]:=x=3\)
    Out[6] \(=3\)
    \(\ln [10]:=f\left[x_{z}\right]:=x^{\wedge} 2\)
    $\ln [11]:=\mathrm{g}\left[\underline{X}, Y_{-}\right]:=\mathrm{f}[\mathrm{X}]+\mathrm{f}[Y]$
$\ln [12]:=g[3,4]$
Out[12] $=25$

## How to Clear Definitions

- When you set a value to a symbol, that value will be used for the symbol for the entire Mathematica session. Assign values to two symbols ( $x$ and $y$ ) and observe their sum:
$\ln [3]:=x=5$;
$y=7 ;$
$x+y$
Out[3]= 12
- Use Clear to clear the definitions for x and y :

Clear[x, y]
■ Observe that x and y no longer have values associated with them:
$\operatorname{In}[5]:=$ Expand $\left[(x+y)^{\wedge} 2\right]$
$\operatorname{Out}[5]=x^{2}+2 x y+y^{2}$

- This command clears all the definitions made during the current Mathematica session:


## Clear["Global"*"]

- Use ClearAll to clear not only the values and definitions of symbols but also the attributes and messages associated with them.

■ You can also use Unset (=.) to clear any values or definitions made to a symbol:
$\ln [20]:=x=5$
Out[20]= 5
$x=$.
?x
■ Remove will remove a symbol completely until it is referenced again:
Remove[x]
?x

## How to Map a Function over a List

- First set up a list of the integers from 1 to 5 :

$$
\begin{aligned}
& \text { In[293]:= mylist = Range[5] } \\
& \text { Out[293]=\{1, 2, 3, 4, } 5\}
\end{aligned}
$$

■ You can map a function over every element of the list using Map;

```
In[294]:= Map[f, mylist]
Out[294]= {f[1], f[2], f[3], f[4], f[5]}
```

- Most mathematical functions have the Listable property:

```
In[297]:= Sin[mylist]
Out[297]= {Sin[1], Sin[2], Sin[3], Sin[4], Sin[5]}
```

■ Map does not just operate on lists. It can be used for any expression:
$\ln [301]:=\mathrm{Map}[\mathrm{f}, \mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}]$
Out[301]= f[a] +f[b] +f[c] +f[d]

- Apply is another functional programming operation. It replaces the head of an expression:
- You can see how this works using two undefined functions, f and g :

In[302]:= Apply[f, g[a, b, c, d]]
Out[302]= f[a, b, c, d]
■ In[304]: = \{Plus[a, b, c], Times[a, b, c], List[a, b, c]\}
Out[304]= \{a + b + c, a b c, \{a, b, c\}\}
■ In[305]:= Apply[Times, a + b + c]
Out[305]= a b c

- Mod finds the remainder when dividing the first number of an ordered pair by the second:
$\ln [307]:=\operatorname{Mod}[10,4]$
Out[307]= 2


## 禺 <br> 



Find

```
Inc18]= my11st = Range [2, 6]
Out18}= {2, 3, 4, 5, 6}
Inc24)- Range [10]
Oul(24]- {1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
Inc25]- Clear[I]
In(27)= Map[f, my11st]
Ou[27]= [I[2], I[3], I[4], I[5], I[6]]
ln(2g)- S1n[my11st]
Oul[28)- [S1n[2],S1n[3], Sin[4], S1n[5], Sin[6]}
Inc291= Map[f,a+b+c+d]
Ou[29]- I[a] + I[b]+I[c] + I[d]
Inc30]- Apply[I, g[a, b, C, d]]
Out(30]= 1[a, b, c, d]
In[31]- Apply[T1mes, a + b + c]
Out(31]= a D c
Inc32]:- Mod[10, 3]
Out[32]= 1
```


## How to Work with Pure Functions

- The most transparent way to define a pure function is with Function. The first argument is a list of arguments, and the second is a function.
$\ln [1]:=\mathrm{f}=$ Function[\{x, y$\}, \mathrm{x}+\mathrm{y}]$
Out[1]= Function[\{x, y\}, x + y]
■ $\ln [2]:=\mathrm{f}[3,4]$
Out[2]= 7
- You do not need to give the function a name to use it:

■ In[3]:= Function[\{x, y\}, $x+y][3,4]$
■ Out[3]= 7
■ $\ln [4]:=\mathrm{g}=(\# 1+\# 2)$ \&
Out[4]= \#1 + \#2 \&
$\ln [5]:=\mathrm{g}[3,4]$
Out[5]= 7

- The advantage of a pure function is that it does not require a separate definition or a name:

```
ln[6]:= (#1 + #2) &[3, 4]
Out[6]= 7
```

■ If the pure function only has one argument, you can use \# instead of \#1. This function squares its argument:
$\ln [7]:=\#^{\wedge} 2 \&[3]$
Out[7]= 9

```
* (a) Untitled-1 *
File Edit Insert Format C_ell Graphics Evaluation Palettes \underline{Window Help}
    ln[[3]:= f= Function[{x, y}, x + y];
    ln[4]:= f[3,4]
    Out[4]= 7
    ln[[]]= (##1 + ##2) &[3,4]
    Out[1]= 7
    ln[2]:= ## ^2&[3]
    Ou[[]= 9

\section*{How to Enter Ranges and Options for Functions}
- Here, \(\operatorname{Sin}[x]\) is the first argument in Plot, while the second argument \(\{x\), \(0,2 \pi\}\) gives the variable and the range for the plot:
\(\operatorname{In}[7]:=\operatorname{Plot}[\operatorname{Sin}[x],\{x, 0,2 \pi\}]\)
Out[7]=

- When plotting functions of two variables, the ranges for each variable are entered in the second and third arguments:
\(\operatorname{In}[6]:=\) Plot3D[Sin[ \(\left.\left.x+y^{\wedge} 2\right],\{x,-3,3\},\{y,-2,2\}\right]\)
Out[6]=

\(\ln [33]:=\operatorname{Plot}[\operatorname{Sin}[x],\{x, 0,2 \pi\}]\)

\(\ln [35]:=\operatorname{Plot} 3 \mathrm{D}\left[\operatorname{Sin}\left[\mathrm{x}+\mathrm{Y}^{\wedge} 2\right],\{\mathrm{x}, 0, \pi\},\{\mathrm{Y}, 0, \pi\}\right]\)


\section*{How to Create Lists}
- Use the shorthand notation \(\}\) to make a list:
\(\ln [1]:=\{2,3,5,6\}\)
Out[1]= \(\{2,3,5,6\}\)
■ Or use List, which automatically is changed to \{\}:
\(\operatorname{In}[2]:=\) List[2, 3, 5, 6]
Out[2]= \(\{2,3,5,6\}\)
- Use Range with one argument to create a list of integers starting at 1:
\(\operatorname{In}[3]:=\) Range[10]
Out[3]=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}
- In[4]:= Range[5, 10]

Out[4]= \(\{5,6,7,8,9,10\}\)
- In[5]:= Range[5, 10, 1/2]

Out[5] \(=\{5,11 / 2,6,13 / 2,7,15 / 2,8,17 / 2,9,19 / 2,10\}\)
- In[6]:= Range[10]^2

Out[6] \(=\{1,4,9,16,25,36,49,64,81,100\}\)
■ In[7]:= Table[i^2, \{i, 10\}]
Out[7]= \(\{1,4,9,16,25,36,49,64,81,100\}\)
■ \(\ln [9]:=\) NestList[f, \(x, 3]\)
Out[9]= \(\{x, f[x], f[f[x]], f[f[f[x]]]\}\)
■ In[10]:= Array[f, 4]
Out[10]= \{f[1], f[2], f[3], f[4]\}
■ Use List to create lists of strings:
■ In[12]:= List[["a", "b", "c"\}, \{"you", "are", "good"\}]
Out[12]= \{\{"a", "b", "c"\}, \{"you", "are", "good"\}\}
- A matrix in Mathematica is a list of lists.
- Use RandomInteger to create a \(4 \times 4\) matrix of random integers between 0 and 10 (stored as m):
- In[13]:= \(\mathrm{m}=\) RandomInteger \([10,\{4,4\}]\)
\(\operatorname{Out}[13]=\{\{5,0,2,5\},\{3,2,8,6\},\{9,9,8,0\},\{2,2,3,4\}\}\)
■ Use MatrixForm to see \(m\) as a 2D matrix:
- MatrixForm[m]

Out[14]//matrixform=
\[
\left(\begin{array}{llll}
5 & 0 & 2 & 5 \\
3 & 2 & 8 & 6 \\
9 & 9 & 8 & 0 \\
2 & 2 & 3 & 4
\end{array}\right)
\]

■ You can directly apply math functions to a list:
\(\operatorname{In}[15]:=\operatorname{Sqrt}[\{1,2,3,4\}]\)
\(\operatorname{Out}[15]=\{1, \sqrt{ } 2, \sqrt{ } 3,2\}\)
\[
\begin{aligned}
& \operatorname{In}[16]:=1+\{\{a\},\{a, b\},\{a, b, c\}\}^{\wedge} 2 \\
& \text { Out[16]=\{\{1+} \left.\left.a^{2}\right\},\left\{1+a^{2}, 1+b^{2}\right\},\left\{1+a^{2}, 1+b^{2}, 1+c^{2}\right\}\right\}
\end{aligned}
\]
- \(\operatorname{In}[17]:=\operatorname{Max[\{ 1,~2,~3,~4\} ]~}\)

Out[17]= 4
\(\operatorname{In}[18]:=\) Length[\{a, b, c\}]
Out[18]= 3
This uses Map to apply Length to each sublist:
\(\operatorname{In}[20]:=\) Map[Length, \(\{\{a\},\{a, b\},\{a, b, c\}\}]\)
Out[20]= \{1, 2, 3\}

\section*{How to Get Elements of Lists}
- \(\operatorname{In}[256]:=\mathrm{v}=\) Range[10]^2

Out[256] \(=\{1,4,9,16,25,36,49,64,81,100\}\)
- \(\operatorname{In}[257]:=\operatorname{Part}[\mathrm{v}, 3]\)
\[
\text { Out[257]= } 9
\]

■ \(\ln [260]:=\mathrm{v}[[5 ; ; 8]]\)
Out[260] \(=\{25,36,49,64\}\)
- \(\operatorname{In}[261]:=v[[-7 ; ;]]\)

Out[261]= \(\{16,25,36,49,64,81,100\}\)
- You can pick out elements from a matrix just like you would from a list.
- In[262]:= m = Partition[Range[25]^2, 5]
\[
\begin{aligned}
\text { Out }[262]= & \{\{1,4,9,16,25\},\{36,49,64,81,100\},\{121,144,169,196,225\},\{256, \\
& 289,324,361,400\},\{441,484,529,576,625\}\}
\end{aligned}
\]
- \(\quad \ln [264]:=m[[1]]\)

Out[264] \(=\{1,4,9,16,25\}\)

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
\(\ln (14):=\mathrm{m}=\) Partition[Range[25] ^2,5]
Out \([4]=\{\{1,4,9,16,25\},\{36,49,64,81,100\},\{121,144,169,196,225\}\), \(\{256,289,324,361,400\},\{441,484,529,576,625\}\}\)
\(\ln \mid 15]\) : \(=\) MatrixForm [m]
Out \([15] /\) MatrixForm \(=\)
\(\left(\begin{array}{ccccc}1 & 4 & 9 & 16 & 25 \\ 36 & 49 & 64 & 81 & 100 \\ 121 & 144 & 169 & 196 & 225 \\ 256 & 289 & 324 & 361 & 400 \\ 441 & 484 & 529 & 576 & 625\end{array}\right)\)

\section*{How to Combine and Rearrange Lists}
\(\operatorname{In}[1]:=\mathrm{v}=\{3,1,3,2,5,4\}\)
\(\operatorname{Out}[1]=\{3,1,3,2,5,4\}\)
\(\operatorname{In}[2]:=\operatorname{Sort}[\mathrm{v}]\)
\[
\operatorname{Out}[2]=\{1,2,3,3,4,5\}
\]

Use Union to sort v and remove any duplicates:
\[
\begin{aligned}
& \operatorname{In}[3]:=\operatorname{Union}[\mathrm{v}] \\
& \operatorname{Out}[3]=\{1,2,3,4,5\}
\end{aligned}
\]
\[
\operatorname{In}[5]:=\mathrm{v}=\text { Union }[\mathrm{v}]
\]
\[
\operatorname{Out}[7]=\{5,4,3,2,1\}
\]
\(\operatorname{In}[11]:=\) Partition[v, 2] \(\operatorname{Out}[11]=\{\{1,2\},\{3,4\}\}\)
\(\operatorname{In}[15]:=\operatorname{Join}[\{\mathrm{a}, \mathrm{b}\},\{\mathrm{c}, \mathrm{d}, \mathrm{e}\},\{\mathrm{f}\), g, h\}]
\(\operatorname{Out}[15]=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{g}, \mathrm{h}\}\)

\section*{How to Perform Operations on Lists}

You can add two lists of the same length element by element:

In[43]:= v = Range[5]
Out[43]= \(\{1,2,3,4,5\}\)
\(\operatorname{In}[49]:=\{\mathrm{a}, \mathrm{b}\}+\{\mathrm{x}, \mathrm{y}\}\)
Out[49]= \(\{\mathrm{a}+\mathrm{x}, \mathrm{b}+\mathrm{y}\}\)
\(\operatorname{In}[50]:=\mathrm{c}+\{\mathrm{a}, \mathrm{b}\}\)
Out[50] \(=\{\mathrm{a}+\mathrm{c}, \mathrm{b}+\mathrm{c}\}\)
\(\operatorname{In}[51]:=\mathrm{k}\{\mathrm{a}, \mathrm{b}\}\)
Out[51]= \{ak, b k \}
\(\operatorname{In}[45]:=\) Append[ \(\mathrm{v}, \mathrm{x}]\)
Out[45] \(=\{1,2,3,4,5, x\}\)
In[44]:= Prepend[v, x]
Out[44] \(=\{x, 1,2,3,4,5\}\)
\(\operatorname{In}[46]:=\operatorname{Insert}[\mathrm{v}, \mathrm{x}, 3]\)
\(\operatorname{Out}[46]=\{1,2, x, 3,4,5\}\)

\section*{How to Create and Use Rules}
- The short form for a rule uses a right arrow, which you get by typing ->

Use /. to use a rule with an expression:
\(\operatorname{In}[57]:=2 x+1 / . x->3\)
Out[57] \(=7\)
- \(\operatorname{In}[59]:=2 \mathrm{x}+\mathrm{y} / .\{\mathrm{x}->3, \mathrm{x}->4\}\)
\(\operatorname{Out}[59]=6+y\)
You can also use a rule to replace larger parts of an expression:
\(\operatorname{In}[62]:=9+x^{\wedge} 2-9 x^{\wedge} 3 / . x^{\wedge} 2-9 x^{\wedge} 3->3 y\)
Out[62] \(=9+3 \mathrm{y}\)
- \(\quad \operatorname{In}[18]:=1+\mathrm{f}[\mathrm{x}]+\mathrm{f}[\mathrm{y}] / . \mathrm{f}[\mathrm{x}-]->\mathrm{x}^{\wedge} 2\)

Out[18]=1 \(+\mathrm{x}^{2}+\mathrm{y}^{2}\)

\section*{How to Use Rule Solutions}
- In[77]: \(=\mathrm{t}=\operatorname{Solve}\left[\mathrm{x}^{\wedge} 2==9, \mathrm{x}\right]\)
\(\operatorname{Out}[77]=\{\{x->-3\},\{x->3\}\}\)
\(\operatorname{In}[2]:=\mathrm{v}=\operatorname{Solve}\left[\left\{\mathrm{x}^{\wedge} 2+\mathrm{y}^{\wedge} 2+\mathrm{z}^{\wedge} 2==9, \mathrm{y}=\mathrm{x}, \mathrm{y}=\mathrm{z}\right\},\{\mathrm{x}, \mathrm{y}, \mathrm{z}\}\right]\)
Out [2] \(=\{\{x->-\sqrt{ } 3, y->-\sqrt{ } 3, z->-\sqrt{ } 3\},\{x->\sqrt{ } 3, y->\sqrt{ } 3, z->\sqrt{ } 3\}\}\)
- \(\mathrm{eqn}=\mathrm{x}^{\wedge} 2+2 \mathrm{ax}+1==0\);
\(\operatorname{In}[97]:=\) sol \(=\) Solve[eqn, x]
Out[97] \(=\left\{\left\{x->-a-\sqrt{ }-1+a^{2}\right\},\left\{x->-a+\sqrt{ }-1+a^{2}\right\}\right\}\)
- \(\operatorname{Plot}[\mathrm{x} /\). sol, \(\{\mathrm{a},-5,5\}]\)

Out[70]=


\section*{Thank You}```

