## **A Brief Introduction to Mathematica**

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# **Outline:**

- Running Mathematica
- Getting Started
- Working with the Notebook Interface
- Getting Information from Mathematica
- The Mathematica System

# **The Structure of Mathematica**

- Mathematica  $\rightarrow$ a modular software system
- The kernel which actually performs computations is separate from the front end which handles interaction with the user.
- To work on Mathematica is to use interactive documents known as notebooks.
- Notebooks mix Mathematica input and output with text, graphics, palettes and other material.
- Notebook interface interactive documents
- Text-based interface text from the keyboard
- MathLink interface

- communication with other programs
- It can interact not only with human users but also with other programs → achieved primarily through MathLink → a standardized protocol for two-way communication between external programs and the Mathematica kernel.

## **Screenshot of a blank notebook**

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# **Using a Notebook Interface**

- A purely graphical interface → double-click the Mathematica icon to start Mathematica.
- A textually based operating system  $\rightarrow$  type the command Mathematica to start.
- An empty notebook with a blinking cursor → by default will interpret your text as input →type Shift+Enter to make Mathematica process your input.
- Shift+Enter tells Mathematica that you have finished your input.

If numeric keypad  $\rightarrow$  its Enter key instead of Shift+Enter.

- After you send Mathematica input from your notebook, Mathematica will label your input with In[n]:=. It labels the corresponding output Out[n]=. Labels are added automatically.
- In[1]:= 2 + 2

Out[1]= 4

notebooks are part of the "front end" to Mathematica.



#### **Getting Information about Mathematica Objects**

#### Log

This gives information on the built-in function Log. Log[z] gives the natural logarithm of z (logarithm to base e). Log[b,z] gives the logarithm to base b.

- Name show information on Name
  - ??Name show extra information on Name
  - ?Aaaa\* show information on all objects whose names begin with Aaaa
- You can put \* anywhere in the string you ask ? about. For example, ? \*Expand would give you all objects whose names end with Expand. Similarly, ?x\*0 would give you objects whose names start with x, end with 0, and have any sequence of characters in between.

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		Log[z] gives the natural logarithm of z (logarithm to base s). Log[b, z] gives the logarithm to base b. $\gg$			
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	In[5]:=	= ?? Export		]	
		Export["file.ext", expr] exports data to a file, converting it to the format corresponding to the file extension ext. Export[file, expr, "format"] exports data in the specified format. Export[file, exprs, elems] exports data by treating exprs as elements specified by elems. >>		2	
	i	Attributes[Export] = {Protected, ReadProtected}			

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# Warnings and Messages

- If it looks as if Mathematica is doing something you definitely did not intend, Mathematica will usually print a message to warn you.
- e.g.the square root function should have only one argument. Mathematica prints a message to warn you that you have given two arguments here.
- In[1]:= Sqrt[4, 5]

During evaluation of In[1]:= Sqrt::argx: Sqrt called with 2 arguments; 1 argument is expected. >>

#### Out[1]= Sqrt[4, 5]

You can suppress the message for one evaluation using Quiet.

In[2]:= Quiet[Sqrt[4, 5]]

Out[2]= Sqrt[4, 5]

# **Interrupting Calculations**

- There will probably be times when you want to stop Mathematica in the middle of a calculation. Perhaps you realize that you asked Mathematica to do the wrong thing. Or perhaps the calculation is just taking a long time, and you want to find out what is going on.
- The way that you interrupt a Mathematica calculation depends on what kind of interface you are using.
- Alt+, notebook interfaces
   Ctrl+C text-based interfaces

# **The Mathematica System**

- Getting Used to Mathematica
- Differences between Computer Systems
- The Limits of Mathematica

#### **Getting Used to Mathematica**

#### Some Important Rules:

Arguments of functions are given in square brackets.

- Names of built-in functions have their first letters capitalized.
- Multiplication can be represented by a space.
- Powers are denoted by ^.
- Numbers in scientific notation are entered, for example, as 0.00025 or 2.5\*10^-4.

Some examples: Sin[x]

Each of these represents multiplication.

a\*b a b a(b+1); 2 x means 2\*x

- Uppercase and lowercase letters are recognized as different characters. Lists are wrapped with curly brackets.
- Commas are used to separate arguments.

#### N[Pi, 50];

## **Differences between Computer Systems**

- Elements of Mathematica that are exactly the same on all computer systems:
- The language used by the Mathematica kernel
- The structure of Mathematica notebooks
- The MathLink communication protocol
- Elements that can differ from one computer system to another:
- The visual appearance of windows, fonts, etc.

Mechanisms for importing and exporting material from notebooks

Keyboard shortcuts for menu commands

Communications between the kernel and the front end are handled by MathLink.

## **The Limits of Mathematica**

- In just one Mathematica command, you can easily specify a calculation that is far too complicated for any computer to do. For example, you could ask for  $Expand[(1+x)^{(10^{100})}]$ . The result of this calculation would have  $10^{100+1}$  terms--more than the total number of particles in the universe.
- If your computer does run out of memory in the middle of a calculation, most versions of Mathematica have no choice but to stop immediately. As a result, it is important to plan your calculations so that they never need more memory than your computer has.
- Memory space is the most common limiting factor in Mathematica calculations. Time can also, however, be a limiting factor. You will usually be prepared to wait a second, or even a minute, for the result of a calculation. But you will less often be prepared to wait an hour or a day, and you will almost never be able to wait a year.

# **Core Language "How to" Topics**

- Create Definitions for Variables and Functions »
- Clear My Definitions »
- Map a Function over a List »
- Work with Pure Functions »
- Enter Ranges and Options for Functions »

## How to Create Definitions for Variables and Functions

Here is a simple transformation rule. It says: whenever you see x, replace it by 3:

```
In[31]:= x = 3
```

```
Out[31]= 3
```

The variable x has a value of 3.

```
In[32]:= x^2
```

```
Out[32]= 9
```

Functions in Mathematica are defined by rules that act on patterns.

```
f[x_] := x^2
```

The rule says: if you have f of any expression, replace it by that expression squared:

```
In[46]:= f[expr]
```

```
Out[46]= expr<sup>2</sup>
```

- g[x\_, y\_] := f[x] + f[y]
   In[49]:= g[3, 4]
   Out[49]= 25
- Always use := to define functions, otherwise the variables on the righthand side may not represent the associated expressions on the left-hand side, since they will be evaluated before the rule is defined.

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ln[6]:= x = 3	ה ונ
Out[6]= 3	
$\ln[10] = f[x_] := x^2$	Ξ
$\ln[11] = g[x_{, y_{}}] := f[x_{}] + f[y]$	Ξ
In[12]:= g[3, 4]	רכ
Out[12]= 25	- LE
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#### **How to Clear Definitions**

When you set a value to a symbol, that value will be used for the symbol for the entire Mathematica session. Assign values to two symbols (x and y) and observe their sum:

In[3]:= x = 5; y = 7; x + y Out[3]= 12

Use Clear to clear the definitions for x and y:

Clear[x, y]

Observe that x and y no longer have values associated with them:
 In[5]:= Expand[(x + y)^2]
 Out[5]= x<sup>2</sup> + 2 x y + y<sup>2</sup>

This command clears all the definitions made during the current Mathematica session:

#### Clear["Global`\*"]

- Use ClearAll to clear not only the values and definitions of symbols but also the attributes and messages associated with them.
- You can also use Unset (=.) to clear any values or definitions made to a symbol:

In[20]:= x = 5 Out[20]= 5 x =.

#### **?**x

Remove will remove a symbol completely until it is referenced again:
 Remove[x]

#### ?x

Information::notfound: Symbol x not found. >>

## How to Map a Function over a List

First set up a list of the integers from 1 to 5:

In[293]:= mylist = Range[5]

- Out[293]= {1, 2, 3, 4, 5}
- You can map a function over every element of the list using Map; In[294]:= Map[f, mylist] Out[294]= {f[1], f[2], f[3], f[4], f[5]}

Most mathematical functions have the Listable property:

In[297]:= Sin[mylist]

Out[297]= {Sin[1], Sin[2], Sin[3], Sin[4], Sin[5]}

Map does not just operate on lists. It can be used for any expression:
 In[301]:= Map[f, a + b + c + d]
 Out[301]= f[a] + f[b] + f[c] + f[d]

- Apply is another functional programming operation. It replaces the head of an expression:
- You can see how this works using two undefined functions, f and g:

```
In[302]:= Apply[f, g[a, b, c, d]]
```

Out[302]= f[a, b, c, d]

In[304]:= {Plus[a, b, c], Times[a, b, c], List[a, b, c]}

Out[304]= {a + b + c, a b c, {a, b, c}}

In[305]:= Apply[Times, a + b + c]

Out[305]= a b c

Mod finds the remainder when dividing the first number of an ordered pair by the second:

```
In[307]:= Mod[10, 4]
Out[307]= 2
```



## How to Work with Pure Functions

The most transparent way to define a pure function is with Function. The first argument is a list of arguments, and the second is a function.

```
In[1]:= f = Function[{x, y}, x + y]
```

Out[1]= Function[{x, y}, x + y]

```
■ In[2]:= f[3, 4]
```

Out[2]= 7

- You do not need to give the function a name to use it:
- In[3]:= Function[{x, y}, x + y][3, 4]
- Out[3]= 7
- In[4]:= g = (#1 + #2) &
  - Out[4]= #1 + #2 &
  - In[5]:= g[3, 4]
  - Out[5]= 7

The advantage of a pure function is that it does not require a separate definition or a name:

```
In[6]:= (#1 + #2) &[3, 4]
Out[6]= 7
```

If the pure function only has one argument, you can use # instead of #1. This function squares its argument:

```
In[7]:= #^2 &[3]
Out[7]= 9
```

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#### How to Enter Ranges and Options for Functions

- Here, Sin[x] is the first argument in Plot, while the second argument {x, 0,2π} gives the variable and the range for the plot:
- In[7]:= Plot[Sin[x], {x, 0, 2π }] Out[7]=



When plotting functions of two variables, the ranges for each variable are entered in the second and third arguments:

In[6]:= Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}] Out[6]=





## **How to Create Lists**

- Use the shorthand notation {} to make a list:
- In[1]:= {2, 3, 5, 6}
- Out[1]= {2, 3, 5, 6}
- Or use List, which automatically is changed to {}:

```
In[2]:= List[2, 3, 5, 6]
```

Out[2]= {2, 3, 5, 6}

- Use Range with one argument to create a list of integers starting at 1:
- In[3]:= Range[10]
- Out[3]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
- In[4]:= Range[5, 10]
- Out[4]= {5, 6, 7, 8, 9, 10}
- In[5]:= Range[5, 10, 1/2]
- Out[5]= {5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2, 10}

In[6]:= Range[10]^2

Out[6]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

In[7]:= Table[i^2, {i, 10}]

Out[7]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

- In[9]:= NestList[f, x, 3]
- Out[9]= {x, f[x], f[f[x]], f[f[f[x]]]}
- In[10]:= Array[f, 4]

Out[10]= {f[1], f[2], f[3], f[4]}

- Use List to create lists of strings:
- In[12]:= List[{"a", "b", "c"}, {"you", "are", "good"}]

Out[12]= {{"a", "b", "c"}, {"you", "are", "good"}}

- A matrix in Mathematica is a list of lists.
- Use RandomInteger to create a 4X4 matrix of random integers between 0 and 10 (stored as m):
- In[13]:=  $m = RandomInteger[10, \{4, 4\}]$

 $Out[13] = \{\{5, 0, 2, 5\}, \{3, 2, 8, 6\}, \{9, 9, 8, 0\}, \{2, 2, 3, 4\}\}$ 

- Use MatrixForm to see m as a 2D matrix:
- MatrixForm[m]Out[14]//matrixform=

$$\begin{pmatrix}
5 & 0 & 2 & 5 \\
3 & 2 & 8 & 6 \\
9 & 9 & 8 & 0 \\
2 & 2 & 3 & 4
\end{pmatrix}$$

You can directly apply math functions to a list: In[15]:= Sqrt[ $\{1, 2, 3, 4\}$ ] Out[15]=  $\{1, \sqrt{2}, \sqrt{3}, 2\}$  In[16]:= 1 + {{a}, {a, b}, {a, b, c}}^2

 $Out[16] = \{\{1 + a^2\}, \{1 + a^2, 1 + b^2\}, \{1 + a^2, 1 + b^2, 1 + c^2\}\}$ 

- In[17]:= Max[{1, 2, 3, 4}]
  - Out[17]= 4
- In[18]:= Length[{a, b, c}] Out[18]= 3
- This uses Map to apply Length to each sublist:
- In[20]:= Map[Length, {{a}, {a, b}, {a, b, c}}]
   Out[20]= {1, 2, 3}

## How to Get Elements of Lists

In[256]:= v = Range[10]^2

Out[256]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

In[257]:= Part[v, 3]

Out[257]= 9

In[260]:= v[[5 ;; 8]]

Out[260]= {25, 36, 49, 64}

In[261]:= v[[-7 ;;]]

Out[261]= {16, 25, 36, 49, 64, 81, 100}

You can pick out elements from a matrix just like you would from a list.

In[262]:= m = Partition[Range[25]^2, 5]

Out[262]= {{1, 4, 9, 16, 25}, {36, 49, 64, 81, 100}, {121, 144, 169, 196, 225}, {256, 289, 324, 361, 400}, {441, 484, 529, 576, 625}}

ln[264]:= m[[1]]

Out[264]= {1, 4, 9, 16, 25}

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#### File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
In[14]:= m = Partition [Range [25] ^ 2, 5]
```

```
Out[14]= {{1, 4, 9, 16, 25}, {36, 49, 64, 81, 100}, {121, 144, 169, 196, 225},
{256, 289, 324, 361, 400}, {441, 484, 529, 576, 625}}
```

\*\*\*\*\*

```
In[15]:= MatrixForm[m]
```

#### Out[15]//MatrixForm=

4

1	1	4	9	16	25
	36	49	64	81	100
	121	144	169	196	225
	256	289	324	361	400
	441	484	529	576	625/

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5]	
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### **How to Combine and Rearrange Lists**

 $In[1]:= v = \{3, 1, 3, 2, 5, 4\}$  $Out[1]= \{3, 1, 3, 2, 5, 4\}$ 

In[2] := Sort[v]

 $Out[2] = \{1, 2, 3, 3, 4, 5\}$ 

Use Union to sort v and remove any duplicates:

In[3]:= Union[v] Out[3]= {1, 2, 3, 4, 5}

In[5]:= v = Union[v]Out[5]= {1, 2, 3, 4, 5} In[7]:= Reverse[v] Out[7]=  $\{5, 4, 3, 2, 1\}$ 

In[11]:= Partition[v, 2] Out[11]=  $\{\{1, 2\}, \{3, 4\}\}$ 

In[15]:= Join[{a, b}, {c, d, e}, {f, g, h}] Out[15]= {a, b, c, d, e, f, g, h}

## **How to Perform Operations on Lists**

- You can add two lists of the same length element by element:
- $In[49]:= \{a, b\} + \{x, y\}$  $Out[49]= \{a + x, b + y\}$  $In[50]:= c + \{a, b\}$  $Out[50]= \{a + c, b + c\}$  $In[51]:= k \{a, b\}$  $Out[51]= \{a k, b k\}$

In[43] := v = Range[5] $Out[43] = \{1, 2, 3, 4, 5\}$ In[45] := Append[v, x] $Out[45] = \{1, 2, 3, 4, 5, x\}$ In[44] := Prepend[v, x]Out[44] = {x, 1, 2, 3, 4, 5} In[46] := Insert[v, x, 3]Out[46] =  $\{1, 2, x, 3, 4, 5\}$ 

## How to Create and Use Rules

- The short form for a rule uses a right arrow, which you get by typing ->
- Use /. to use a rule with an expression:
- In[57] := 2 x + 1 / x -> 3

Out[57]= 7

In[59]:=  $2 x + y /. \{x \rightarrow 3, x \rightarrow 4\}$ 

Out[59] = 6 + y

 You can also use a rule to replace larger parts of an expression: In[62]:= 9 + x^2 - 9 x^3 /. x^2 - 9 x^3 -> 3 y
 Out[62]= 9 + 3 y
 In[18]:= 1 + f[x] + f[y] /. f[x\_] -> x^2
 Out[18]= 1 + x<sup>2</sup> + y<sup>2</sup>

#### How to Use Rule Solutions



# **Thank You**