Simulating a coin toss experiment using Mathematica

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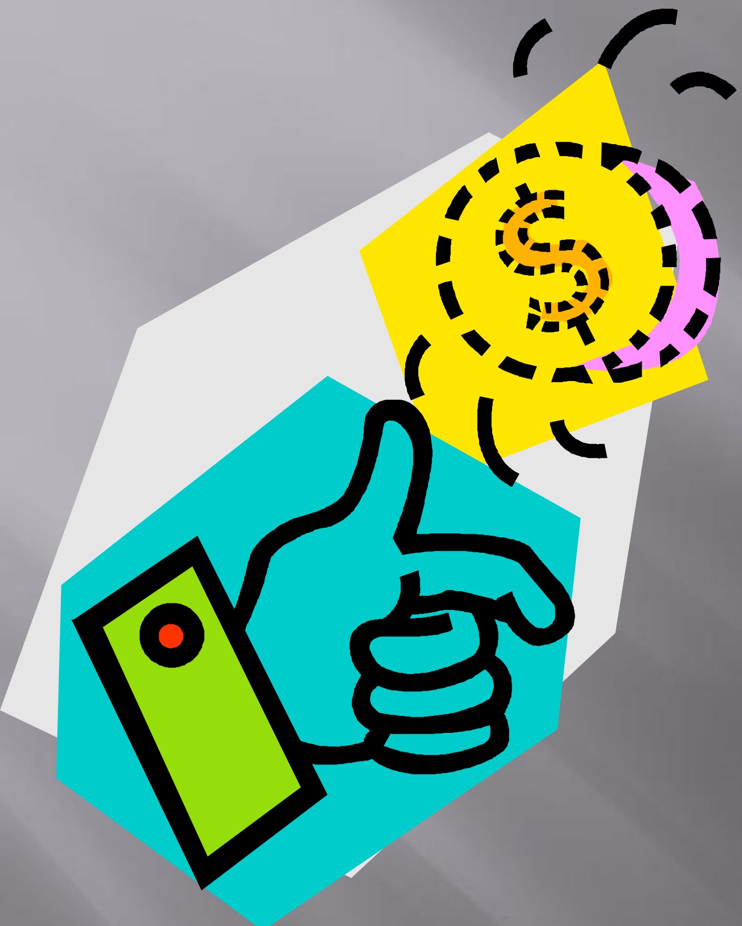


Consider a virtual experiment of tossing a coin 5 times

- Objectives:
- Getting the experimental frequency distribution using random numbers
- Studying the theoretical probability distribution
- Comparing theory versus experiment: chi-square minimization

Random Chices

- RandomChoice[{a,b,c},n]
- This command randomly selects n objects from the list {a,b,c}. For every new run of the command, we will get a different result just like a true random experiment….

```
In[2]:= RandomChoice[{0, 1, 3}, 6]

Out[2]= {1, 3, 3, 1, 0, 0}
```

Coin toss: Randomly choose one number from 0 and 1.

0 corresponds to **Tail** and 1 corresponds to **Head**

Repeat this 5 times: this will constitute one run of the experiment. Let's call it a run/coin…

Make this variable coin dynamical variable. Every time one calls a dynamic variable, it is evaluated afresh.

coin:=RandomChoice[{0,1},5]

```
In[15]:=
    coin := RandomChoice[{0, 1}, 5]
    coin

Out[16]= {1, 1, 0, 1, 1}
```

To calculate number of heads in an experiment we simply add elements of one experiment. The variable head is also made dynamic.

Any expression followed by the // operator and a function name is equivalent to that function applied on that expression.

expression // function = function[expression]

Next we want to repeat this experiment n times. This is done by tabulating/looping the dynamic variable head n times. Call this function "repetition" of input variable n.

Let's do 10 runs of the experiment as an example.

```
In[57]:= heads := coin // Total
         repetition[n_] := Table[heads, {i, 1, n}]
         repetition[10]
Out[59]= {2, 3, 3, 1, 4, 1, 2, 0, 2, 4}
```

A run of the experiment can result into 0, 1,2 ,3 ,4 or 5 heads.

1000 runs of the experiment will result into an array of {0,1,2…5} in which 0 comes f0 times, 1 comes f1 times, …, 5 comes f5 times.

The array {f0,f1,f2,….f5} is called the frequency distribution.

This mathematica command **Tally** generates an unsorted array {{1,f1}, {2,f2}, {3,f3} ,…, {5,f5}}

This array can be sorted using the Sort command.

```
In[119]:= Tally[repetition[1000]]
          Sort[Tally[repetition[1000]]]
          array = Sort[Tally[repetition[1000]]];
Out[119]= {{2, 321}, {5, 20}, {3, 298}, {1, 161}, {4, 166}, {0, 34}}
Out[120]= {{0, 24}, {1, 156}, {2, 299}, {3, 341}, {4, 144}, {5, 36}}
```

Plotting Frequency Distribution

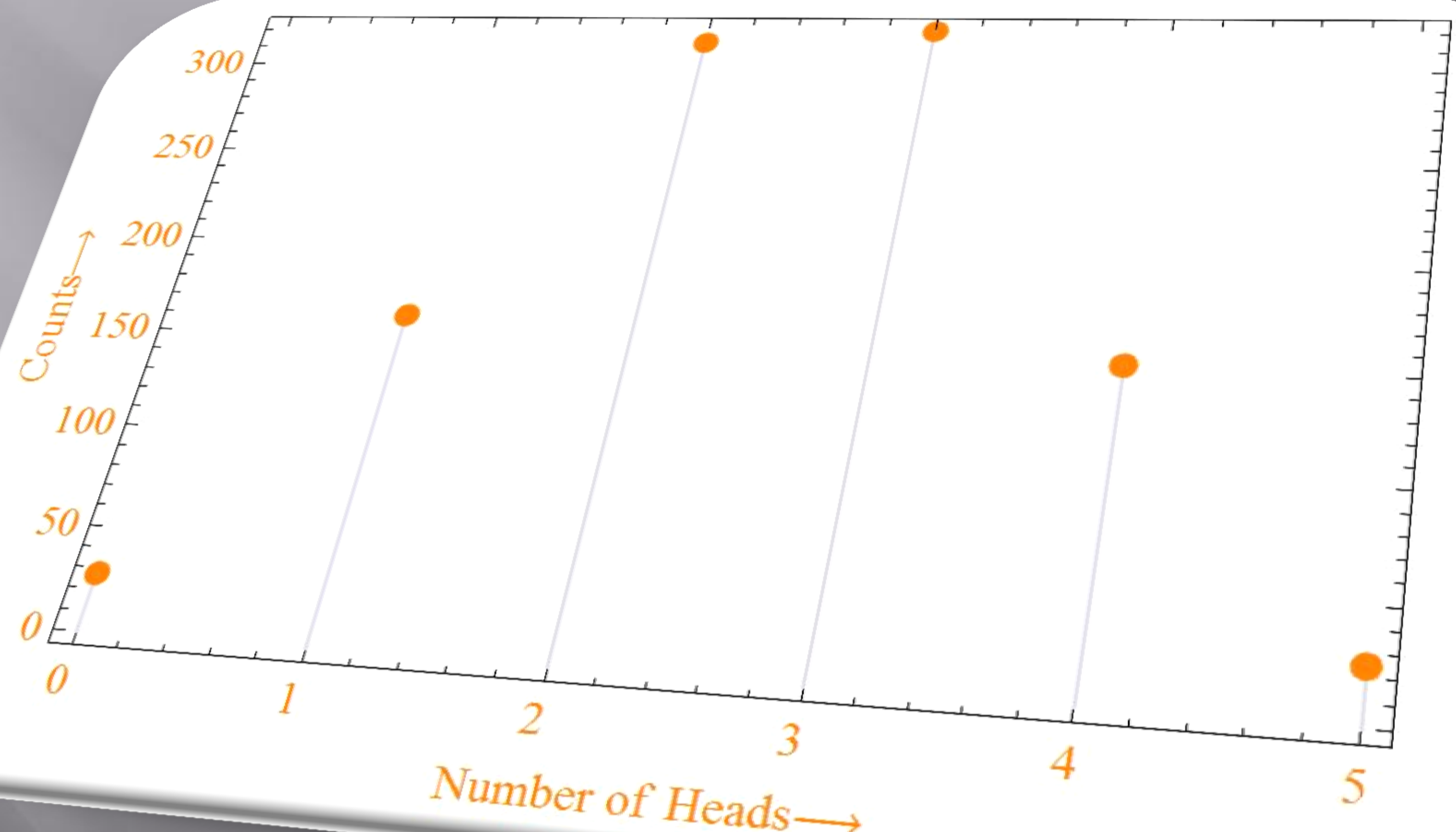
To plot frequency distribution we will use mathematica command ListPlot:-

freqdis=ListPlot[array,Filling->Axis,Frame->True,FrameLabel->{"Number of Heads\[LongRightArrow]","Counts\[LongRightArrow]"},PlotStyle->Directive[PointSize[Large],Orange],LabelStyle->Directive[Medium,Orange]]

- **Filling is an option to** specifies what filling to add under points, curves and surfaces.
- **Frame** option decide weather we want a frame on plot or not.
- **FrameLabel** specifies labels to be placed on the edges of a frame.
- **PlotStyle** specifies styles in which objects are to be drawn.

```
In[8]:=
freqdis = ListPlot[array, Filling → Axis, Frame → True,
  FrameLabel → {"Number of Heads⟶", "Counts⟶"}, PlotStyle → Directive[PointSize[Large],
   Orange], LabelStyle → Directive[Medium, Orange]]
```

Experimental Frequency Distribution



Plotting Probability Distribution

Probability of getting 1 head in 1000 runs of experiment is given by = (Number of runs of experiment giving 1 head as output)÷(Total runs of experiment).

To get Probability Distribution we will divide frequencies of each outcome by total number of runs i.e 1000 in our case. Mathematica command MapAt will be used

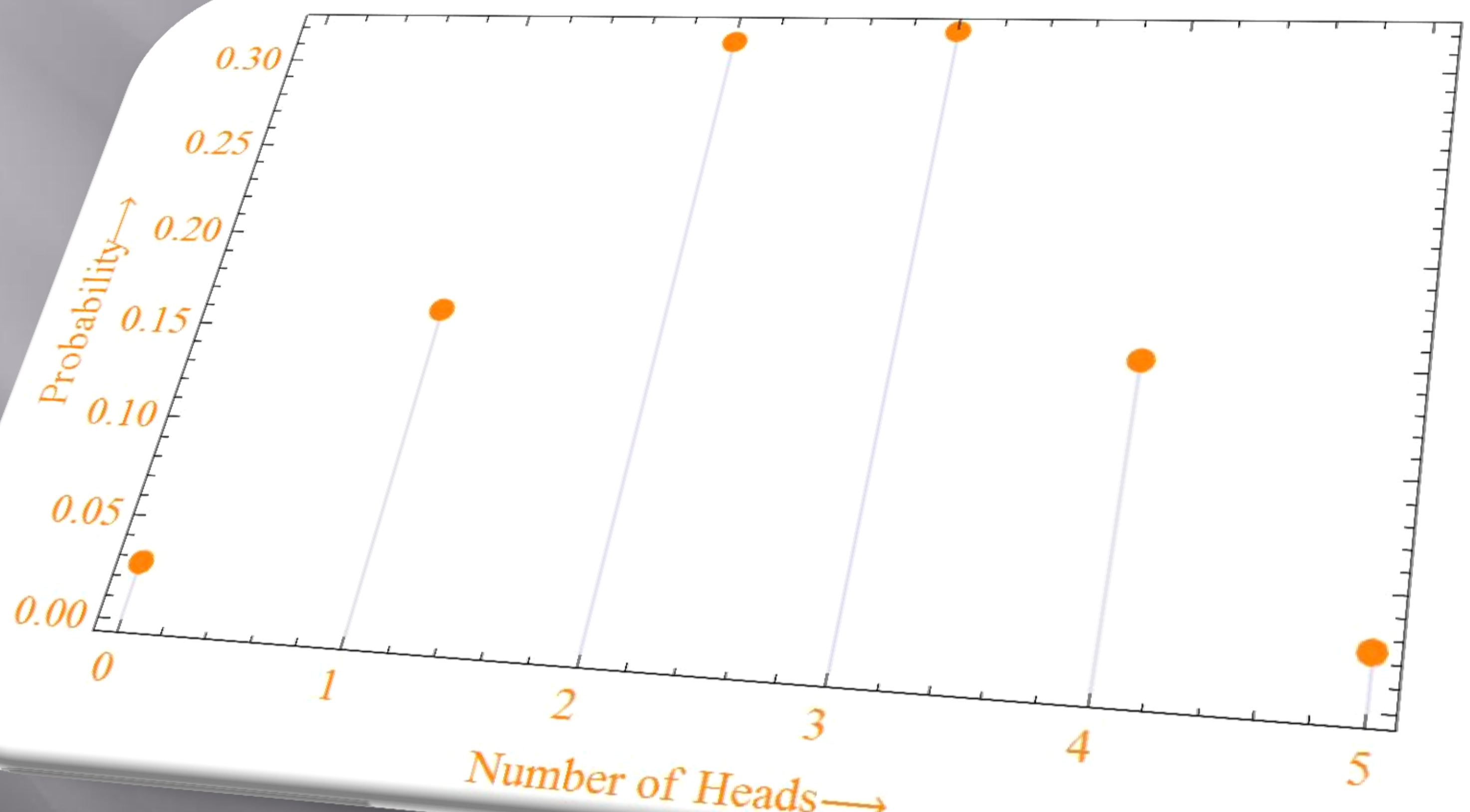
prob=MapAt[(#/1000)&,array,Table[{i,2},{i,1,6}]]

To plot Probability Distribution **PlotList** will be used again

expprob=ListPlot[prob,Filling->Axis,PlotStyle->Directive[PointSize[Large],Orange]]

```
prob = MapAt[(#/1000) &, array, Table[{i, 2}, {i, 1, 6}]];
expprob = ListPlot[prob, Filling -> Axis, PlotStyle → Directive[PointSize[Large],
   Orange], Frame -> True, FrameLabel → {"Number of Heads⟶", "Probability⟶"},
LabelStyle → Directive[Medium, Orange]]
```

Experimental Probability Distribution



Theoretical Overview

Outcome of coin tosses follows Binomial Distribution which can be verified as follows: -

Consider a toss of 3 coins. Sample space is
{{HHH}{HHT}{HTH}{THH}{TTH}{THT}{HTT}{TTT}}
Probability of 0 head = 1/8
Probability of 1 head = 3/8
Probability of 2 head = 3/8
Probability of 3 head = 1/8

Binomial = $\binom{n}{k} p^k q^{n-k}$

$\binom{3}{0}(1/2)^0 (1/2)^3 = 1/8$ [0 Head]

$\binom{3}{1}(1/2)^1 (1/2)^2 = 3/8$ [1 Head]

$\binom{3}{2}(1/2)^2 (1/2)^1 = 3/8$ [2 Head]

$\binom{3}{3}(1/2)^3 (1/2)^2 = 1/8$ [3 Head]

For Binomial Distribution we will be using following mathematica commands

BinomialDistribution[n,p]

- represents a binomial distribution with n trials and success probability p.

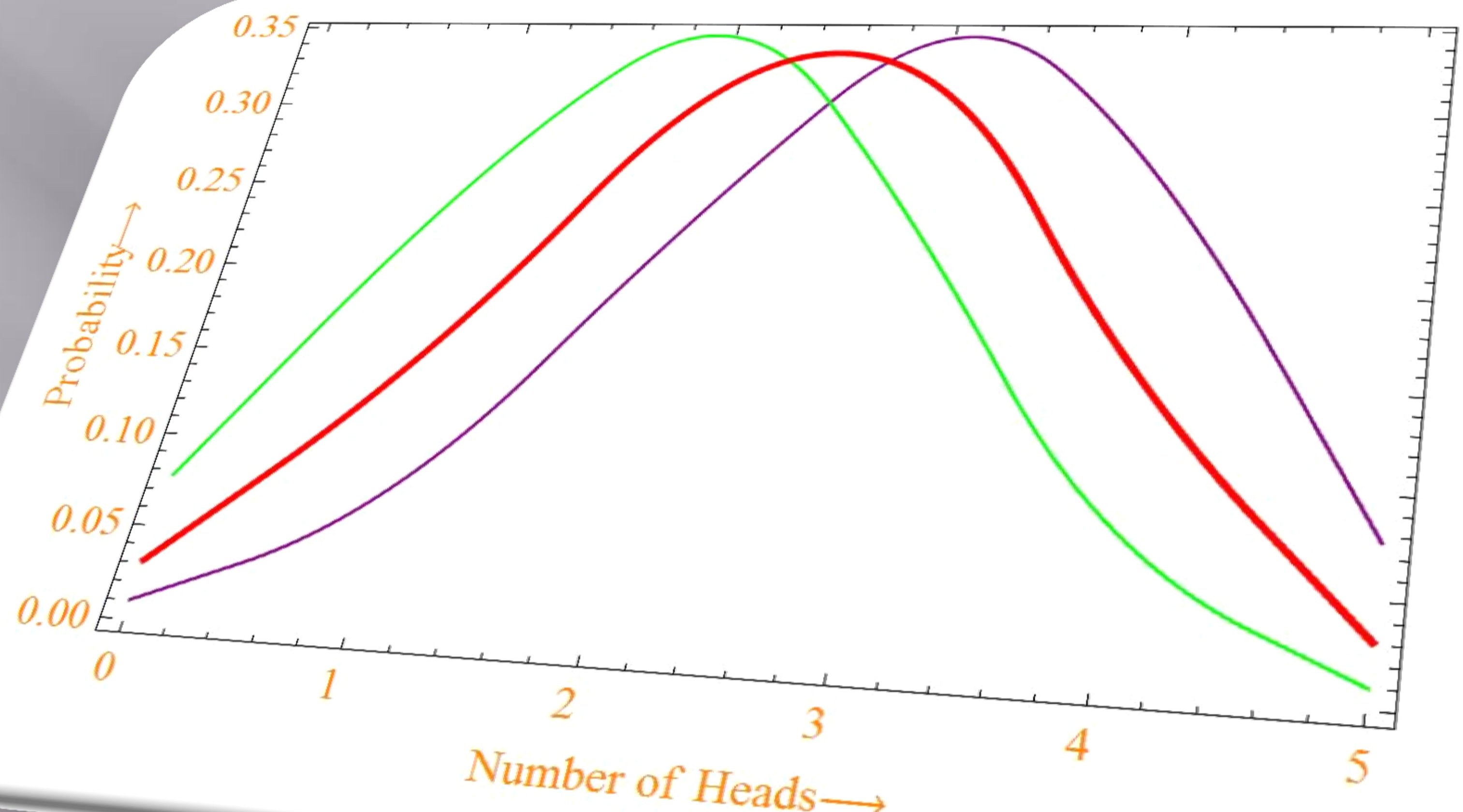
PDF[BinomialDistribution[n,p],x]

- gives the probability density function for the Binomial distribution evaluated at x.

We will plot three graphs corresponding to probabilities 0.4,0.5 and 0.6 . To show these three graphs on a single graph for comparison we will use Mathematica Command **Show**

```
th1 = ListPlot[Table[{x, PDF[BinomialDistribution[5, 0.5], x]}, {x, 0, 5, 1}], InterpolationOrder → 2, Joined → True, AxesOrigin → {0, 0},
  PlotStyle → Directive[Red, Thick]]
th2 = ListPlot[Table[{x, PDF[BinomialDistribution[5, 0.4], x]}, {x, 0, 5, 1}], InterpolationOrder → 2, Joined → True, AxesOrigin → {0, 0},
  PlotStyle → Directive[Green]]
th3 = ListPlot[Table[{x, PDF[BinomialDistribution[5, 0.6], x]}, {x, 0, 5, 1}], InterpolationOrder → 2, Joined → True, AxesOrigin → {0, 0},
  PlotStyle → Directive[Purple]]
Show[th3, th2, th1, Frame → True, FrameLabel → {"Number of Heads⟶", "Probability⟶"}, LabelStyle → Directive[Orange, Medium]]
```

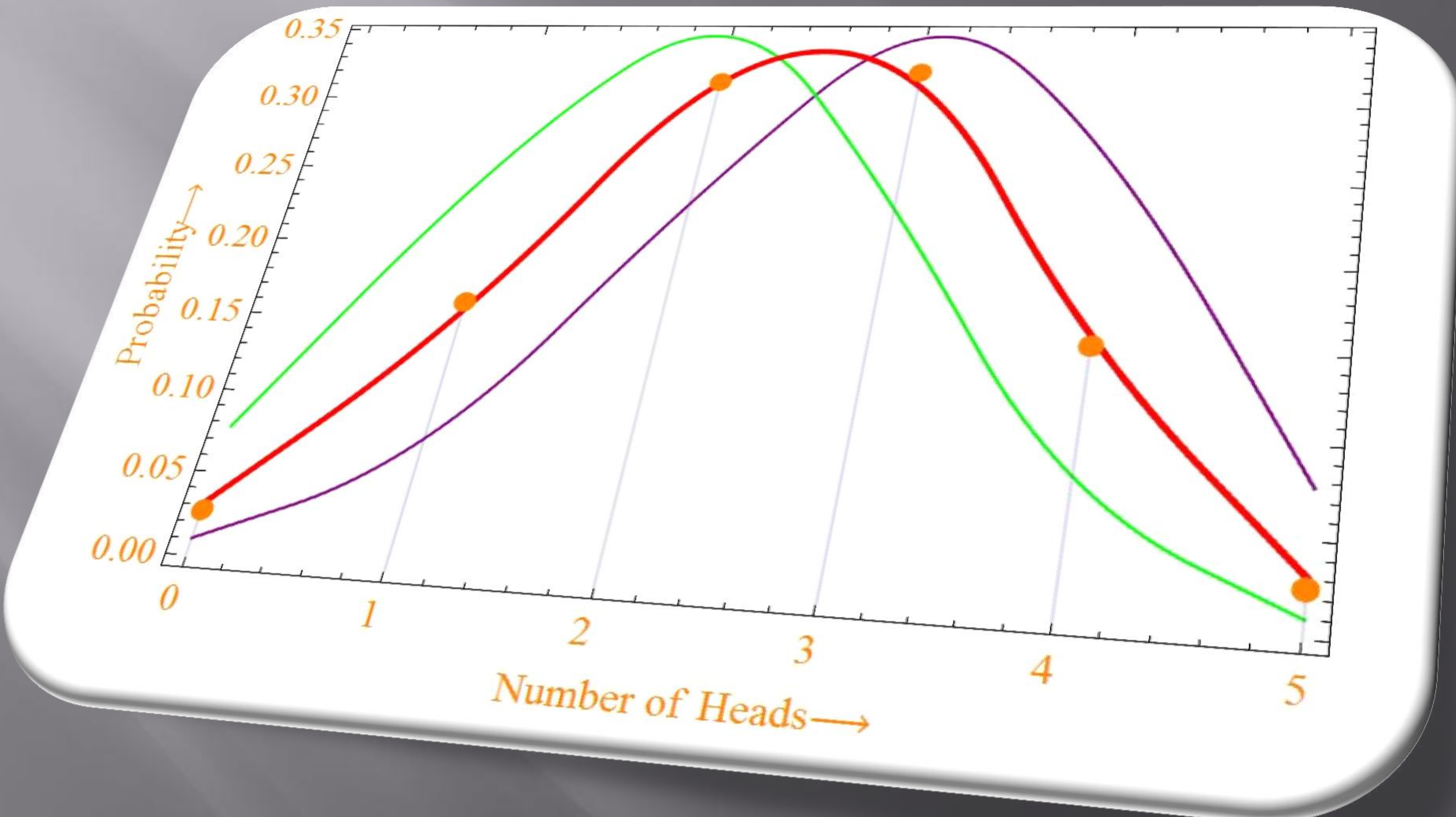
Theoretical Probability Distributions



Experiment

VS

Theory

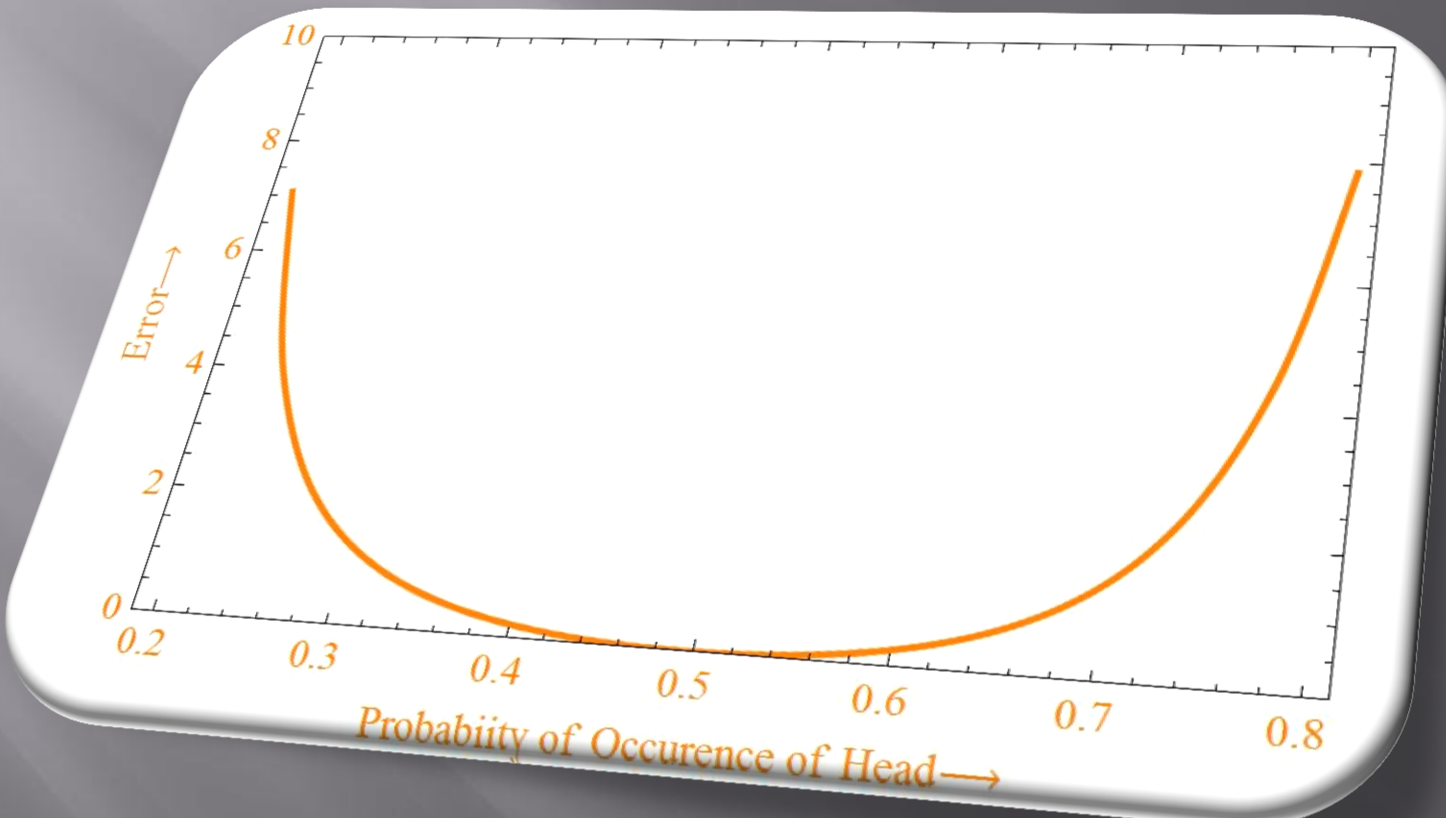


Chi Square Test

- Basically in Chi square we will be evaluating the formula $\sum_{k=1}^{n}(Ek\text{-}T_k)^2/T_k$

```
In[58]:= thx[m_] := Table[{x, PDF[BinomialDistribution[5, m], x]}, {x, 0, 5, 1}];
subt[m_] := thx[m] - prob
neum[m_] := Table[subt[m][[i, 2]], {i, 1, 6}];
denm[m_] := Table[thx[m][[i, 2]], {i, 1, 6}];
chi[m_] := (neum[m])^2/denm[m];
chisq[m_] := Total[chi[m]];
tabchisq = Table[{i, chisq[i]}, {i, 0.2, 0.8, 0.02}];
ListPlot[tabchisq, Joined → True, PlotRange → {0, 10}, InterpolationOrder → 2, Frame → True, FrameLabel → {"Probabiity of Occurence of Head→", "Error→"},
LabelStyle → Directive[Orange, Medium], PlotStyle → Directive[Thick, Orange]]
```

Error Plot



10
8
6
4
2
0
Error⟶
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Probabiity of Occurence of Head⟶

- Error Plot has minima at probability of 0.5 That means probability of occurrence of one head is 0.5 which is TRUE.
- Results obtained by a simulation in mathematica are same as a real experiment.
- So our virtual coin is behaving like a real coin.

Conclusions

Thank You.