TOPIC : Matrix Manipulation

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MATRIX :

Matrix can be define as two dimentional orderly arrangement of data. Data can be anything say no.s, symbols or anythings which are of the same type. since matrix are two dimentionl array, it has no. of row and coulums which are called dimention of the matrix. In case of square matrix it is called as order of the matrix.

Matrix behave like numerical no.s and perform mathematical algebra such as addition , substractions, multiplication and division, but they follows certain rules. i,e
For addition :- Addition of matrix take place only when the concern matrixes have same dimention and addition happen only amongs the elements which have same position. Example ;

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 5 & 0 \\ 6 & 0 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 0 \\ 0 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 6 \\ 4 & 5 & 0 \\ 6 & 8 & 18 \end{pmatrix} \tag{1}$$

For multiplication ;- Multiplication of matrix take place only when the no.s of coulums of the first matrix is equal to the no.s of row of the second matrix. here the element of the resultant matrix is given by

$$x_{ij} = \sum_k x_{ik} * x_{kj} \tag{2}$$

Example ;

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 5 & 0 \\ 6 & 0 & 9 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 0 \\ 0 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 26 & 30 \\ 20 & 0 & 0 \\ 6 & 84 & 99 \end{pmatrix} \tag{3}$$

In matrix multiplication we have two importain multiplication, i,e inner product and outer product.

Inner product ; It is define as $A * B = A^T.B$ i,e product of two vector gives a scalar.

Example ;

$$\begin{pmatrix} 6 & 0 & 9 \end{pmatrix} \times \begin{pmatrix} 0 \\ 8 \\ 9 \end{pmatrix} = 81 \quad (SCALAR) \tag{4}$$

Outer product ; It is define as $A * B = A.B^T$ i,e product of two vector give out an operator, i, e a square matrix.

Example ;

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \tag{5}$$

Matrix have different types according to their numbers of rows and coulums, i,e we have Row martix, coulum matrix, Null matrix, ractangular matrix and square matrix.

Square matrix is very importain in study the physical nature of this world.It has different types depending upon their properties.we have Diagonal matrix, Identity matrx, Symetric matrix,skew symetrix, hermatian matrix, skew hermatian matrix, Tringular matrix Orhtogonal matrix,unitary matrix,singular matrix,idempodent matrix and involuntary matrix etc.

Square matrix has been taking very importain role in physics and mathematical problem analysis. It is usefull because most of the properties associate with it can be express in term of some inportain quantity. say Trace, determinent, Rank, Eigen Value and Eigen Vector Problems etc.

Rank of a squre matrix ; Rank is define as the number of linearly independent no.s of rows and coulums of the square matrix.

Let us consider a singular matrix having order n, its determinent is $\nabla = 0$. we can transform it into echelon form and found a new square matrix whose determinent $\nabla \neq 0$, after removing the row or coulum which depend on the other rows and coulums. thus the maximum no. of rows or couloms of the newly found new matrix is define as Rak of the matrix. let the no. of removed no.s of row or coulums be u, then the rank of the matrix is given by $r = n - u$.

Trace of a squre matrix ; Trace of a square matrix is define as the sum of all the diagonal elements of the matrix. It is also equal to the sum of all the eigen values of the matrix.

Determinent of a square matrix ;

It is define as a unique number which gives many properties of the concern matrix. Say, it determine the nature of the roots, whether singular or not etc. In mathematics, determinent is given by,

$$\nabla = \sum_j a_{1j}.C_{1j}. \tag{6}$$

where C_{1j} is the co-factor of a_{1j}.

There are many properties of Determiment of square matrix, say A i.e.

1. A and A^T have same determinent ∇.
2. A is singular if its determinent is equal to zero i,e $\nabla = 0$.
3. If two consecutive row or coulum are interchange, then its sign changes by -1 times the no. of changing.
4. Determinent of a matrix is equal to the product of the eigen value of the matrix.

Use of Determinent;

◇ **In Co-ordinate Geometry;**

Using Determinent concepts, we can determine following parameters. i.e.

1. The area of a tringle from its co-ordinate points of the vertex. thus area ia given by

Area of the triangle is

$$\nabla = \frac{1}{2} \times \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{7}$$

2. The equation of straight line is given by

$$\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \tag{8}$$

The determinent concept is use for checking the condition for collinearity of points on a plane.

◇ **Solution of linear System of equation;**

For a system of Linear System of Equation, Such as

$a_1x_1 + b_1y_1 + c_1z_1 = r_1$
$a_1x_2 + b_1y_2 + c_1z_2 = r_2$
$a_1x_3 + b_1y_3 + c_1z_3 = r_3$

we can express it in the form of matrix Equation, i,e $AX = Y$ thus, we get

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \times \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \tag{9}$$

these$AX = Y$, type of equation can be solved by
A. Matrix Methods gives $X = A^{-1}Y$

i,e

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}^{-1} \times \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \tag{10}$$

B. Cramers rules gives

$$x_1 = \frac{\begin{vmatrix} r_1 & b_1 & c_1 \\ r_2 & b_2 & c_2 \\ r_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \tag{11}$$

$$x_2 = \frac{\begin{vmatrix} a_1 & r_1 & c_1 \\ a_2 & r_2 & c_2 \\ a_3 & r_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \tag{12}$$

$$x_3 = \frac{\begin{vmatrix} a_1 & b_1 & r_1 \\ a_2 & b_2 & r_2 \\ a_3 & b_3 & r_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \tag{13}$$

Not only these there are many application of Determinent in Science, say Nature of the root of system of linear equations, Linear Homogenous Equations can also determind from the determinent of their co-efficient.

Eigen Value and Eigen Vector Problems; Square matrixes satisfied eigen values eqns which has been taking importain roles in Quantum Mechanics and many othe Problems solving cases.Thus Matrix has its characteristic eqns of the form $A|\psi\rangle = \lambda|\psi\rangle$ solving it we can determind its Eigen values and its corresponding Eigen Vectors.
Thus in Two Dimensions we have

For $A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ $|A - \lambda I| = 0$
It can be express in the form of Quadratic eqns of the form
$\Rightarrow a\lambda^2 + b\lambda + c = 0$
where $a = 1, b = trace, c = a_1b_2 - a_2b_1$
Root of this Eqn will be the requred eigen values.
For solving eigen values,we should check the nature of the Discriminent. i, e
$D = b^2 - 4ac$
if $D >= 0$ then there will be real root,
but if $D < 0$, then there will be no real root.
thus the root is given by

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{here +sign for root 1, and -ve for the other root.} \tag{14}$$

In Three Dimensions we have

for $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ $|A - \lambda I| = 0$

It can be express in the form of cubic eqns of the form
$\Rightarrow a\lambda^3 + b\lambda^2 + c\lambda + d = 0$

where $a = -1$, $\quad b = trace$, $c = a_2b_1 + a_3c_1 + b_3c_2 - a_1b_2 - a_1c_3 - b_2c_3$
$d = (a_1*b_2*c_3 - a_1*c_2*b_3 - a_2*b_1*c_3 + a_2*c_1*b_3 + a_3*b_1*c_2 - a_3*b_2*c_1)$;
Root of this Eqn will be the requred eigen values.
For solving eigen values,
we should check the nature of the Discriminent. i, e
$D = b^2c^2 - 4ac^3 - 4db^3 - 27a^2d^2 + 18abcd$;
if $D >= 0$ then there will be real root,
but if $D < 0$, then there will be no real root.

thus for finding the root, we define...

$$x = \frac{3c/a - b^2/a^2}{3} \tag{15}$$

$$y = \frac{(2b^3/a^3 - 9bc/a^2 + 27d/a)}{27} \tag{16}$$

$$z = \frac{y^2/4 + x^3}{27} \tag{17}$$

$$y1 = \sqrt{(y^2)/4 - z} \tag{18}$$

$$y2 = y1^{1/3} \tag{19}$$

$$y3 = \cos -y/2y1 \tag{20}$$

$$y4 = \cos y3/3 \tag{21}$$

$$y5 = \sqrt{3 \sin y/3} \tag{22}$$

$$y6 = -b/3a \tag{23}$$

thus the required E- values of the squre matrix are...

$E1 = 2y2y4 + y6$

$E2 = -y2 \times y4 - y5 + y6$

$E3 = -y2 \times y4 + y5 + y6$

CONCLUSION;

After knowing all these e-values, we can determine all the eigan vectors.THus we can confirme wether the the set of Eigen vectors are linearly independent or not testing $det = 0$ .Problems of Quantum mechanics which can be manipulated in term of eigen values and eigen vectors become easily solved.Concerning Heisenberg matrix mechanicsof discrete basis, all the operator, observable are represent by square matrix. State vectorsby rows and couloms matrixes and many properties are analyse through eigen value/eigen vector problems. thus we can studysimple harmonic eqns, angular momentum, unitary transformation so easily by matrix manipulations.
Comutation properties or operators are also easily solved in matrix methods. Not only these Matrix concept is use in verious fields of science. In electronics it help in complex circuit analysis Using$RI = V$. In four dimention space-time relativistic concept, Tensor analysis of multidimentional theory, Matrix concept is very importain.

Thus concerning all these advantage and application of Matrix concepts, I should say Matrix Manipulation is very importain to Science.We can also set up verious programe to perform these Matrix Manipulation. Thus i am demonstrating some simple programe say...
1. C++ rpograme for Matrix Multiplication of any dimentions.
2. C++ rpograme for finding Determinent and Trace of Matrix
3. C++ rpograme for finding eigen value and eigen vectors of square matrix upto order three.etc.
these ere very helpfull to solved many problems of physics by saving lots of times.

Thank you