Data Sampling & Nyquist Theorem

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Signal Processing :

Signal :

Any physical quantity that varies with time, space, or any other independent variable or variables. Signals categorizes to the fields of communications, signal processing, and to electrical engineering more generally. Within a complex society, any set of human information or machine data can also be taken as a signal.

Classification of Signals :

- Continuous -Time Signals
- Discrete -Time Signals
- Continuous -Valued Signals
- Discrete- Valued Signals

Continuous -Time Signals :

- ➢ defined for every value of time
- ➢ take on values in continuous interval (a , b),where a can be -∞ and b can be ∞.
- ➢ can be described by functions of a continuous variables

Discrete -Time Signals :

- ➢ defined only at certain specific values of time
- ➢ time instants need not be equidistant, but in practice they are usually taken at equally spaced intervals

The values of a continuous-time or discrete-time signal can be continuous or discrete.

Continuous-Valued Signals :

If a signal takes on all possible values on a finite or an infinite range it is said to be continuous-valued signals.

Discrete-Valued Signals :

If a signal takes on values from a finite set of possible values,it is said to be a discrete-valued signals.

Signal Processing:

Signal processing is an area of systems engineering, electrical engineering and applied mathematics that deals with operations on or analysis of signals, in either discrete or continuous time. Signals of interest can include sound, images, time-varying measurement values and sensor data, for example biological data such as electrocardiograms, control system signals, telecommunication transmission signals, and many others. Signals are analog or digital electrical representations of time-varying or spatial-varying physical quantities.

Typical Operations And Applications :

Processing of signals includes the following operations and algorithms with application examples:

1. Filtering (for example in tone controls and equalizers)
2. Smoothing, deblurring (for example in image enhancement)
3. Adaptive filtering (for example for echo-cancellation in a conference telephone, or denoising for aircraft identification by radar)
4. Spectrum analysis (for example in magnetic resonance imaging, tomographic reconstruction and OFDM modulation)
5. Digitization, reconstruction and compression (for example, image compression, sound coding and other source coding)
6. Storage (in digital delay lines and reverb)
7. Modulation (in modems and radio receivers and transmitters)
8. Wavetable synthesis (in modems and music synthesizers)
9. Feature extraction (for example speech-to-text conversion and optical character recognition)
10. Pattern recognition and correlation analysis (in spread spectrum receivers and computer vision)
11. Prediction
12. A variety of other operations

Categories of signal Processing :

- **Digital signal processing:**

Digital signal processing (DSP) is concerned with the representation of discrete time, discrete frequency, or other discrete domain signals by a sequence of numbers or symbols and the processing of these signals. Digital signal processing and analog signal processing are subfields of signal processing. DSP includes subfields like: audio and speech signal processing, sonar and radar signal processing, sensor array processing, spectral estimation, statistical signal processing, digital image processing, signal processing for communications, control of systems, biomedical signal processing, seismic data processing, etc.

The goal of DSP is usually to measure, filter and/or compress continuous real-world analog signals. The first step is usually to convert the signal from an analog to a digital form, by sampling and then digitizing it using an analog-to-digital converter (ADC), which turns the analog signal into a stream of numbers. However, often, the required output signal is another analog output signal, which requires adigital-to-analog converter (DAC).

- **Analog signal processing:**

Analog signal processing is for signals that have not been digitized, as in legacy radio, telephone, radar, and television systems. This involves linear electronic circuits such as passive filters, active filters, additive mixers, integrators and delay lines. It also involves non-linear circuits such ascompandors, multiplicators (frequency mixers and voltage-controlled amplifiers), voltage-controlled filters, voltage-controlled oscillators and phase-locked loops.

Advantages of Digital over Analog Signal Processing

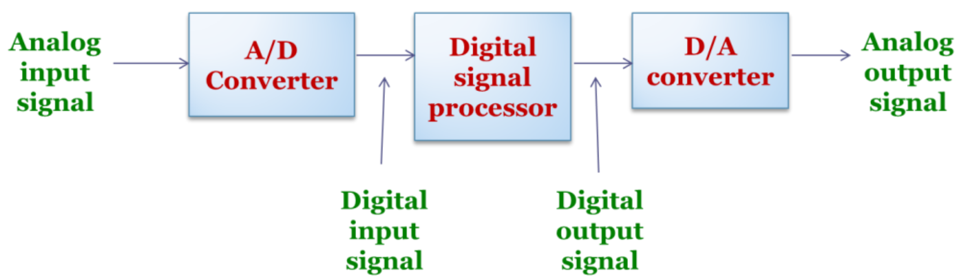
- Digital system can be simply reprogrammed for other applications/ported to different hardware / duplicated
- Reconfiguring analog system means hardware redesign, testing, verification
- DSP provides better control of accuracy requirements
- Analog system depends on strict components tolerance, response may drift with temperature
- Digital signals can be easily stored without deterioration
- Analog signals are not easily transportable and often can’t be processed off-line
- More sophisticated signal processing algorithms can be implemented
- Difficult to perform precise mathematical operations in analog form

Digital Signal Processing :

For a signal to be processed digitally,

- it must be discrete in time
- Its values must be discrete

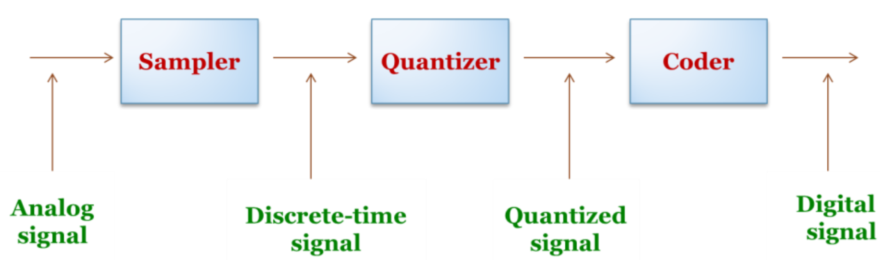
Block diagram of a digital signal processing system



An analog-to-digital converter (abbreviated ADC,A/D or A to D) is a device that converts a continuous quantity to a discrete time digital representation. An ADC may also provide an isolated measurement. The reverse operation is performed by a digital-to-analog converter (DAC).

A/D conversion is a three-step process :

Step 1 : Sampling
Step 2: Quantization
Step 3: Coding



Step 1 : Sampling of Analog signal

Conversion of continuous-time signal into a discrete-time signal by taking samples of continuous-time signal at discrete time instants.

A continuous time sinusoidal signal is :

$$\mathbf{x_a(t) = A\cos(\Omega t + \theta) \ , \ -\infty < t < \infty} \qquad (1)$$

Where,

$x_a(t)$: an analog signal

A : is amplitude of the sinusoid

Ω :is frequency in radians per seconds(rad/s)

θ : is the phase in radians

$$\mathbf{\Omega = 2\pi F}$$

A discrete-time sinusoidal signal obtained by taking samples of the analog signal $x_a(t)$ every T seconds may be expressed as

$$\mathbf{x(n) = x_a(nT) = A\cos(\omega n + \theta) \ , \ -\infty < n < \infty} \qquad \mathbf{(2)}$$

Where,

n : an integer variable, called sample number

A : is amplitude of the sinusoid

ω : frequency radians per sample

θ : is the phase in radians

$$\mathbf{\omega = 2\pi f}$$

f : frequency cycles per samples

T is the **sampling interval** or **sampling period**

$F_s = \frac{1}{T}$ is called the **sampling rate** or the **sampling frequency** Hertz)

- **Relationship b/w frequency of analog and digital signal is**

$$\mathbf{f = \frac{F}{Fs}}$$

- **Range of frequency variables**

$$-\infty < F < \infty$$

$$-1/2 < f < ½$$

- Frequency of the continuous-time sinusoid when sampled at rate F_s must fall in the range

$$-\frac{Fs}{2} \leq \mathbf{F} \leq \frac{Fs}{2}$$

- The highest frequency in the discrete signal is $f = \frac{1}{2}$,
- With a sampling rate F_s , the corresponding highest value of F is

$$F_{max} = \frac{Fs}{2}$$

$$\Downarrow$$

Sampling introduces anambiguity

Limitations of DSP – Aliasing:

Most signals are analog in nature, and have to be sampled

$\Rightarrow$ **loss of information**

- we only take samples of the signals at intervals and don't know what happens in between

$\Rightarrow$ **aliasing**

cannot distinguish between higher and lower frequencies

Sampling theorem: to avoid aliasing, sampling rate must be at least twice the maximum frequency component (`bandwidth') of the signal

- To avoid ambiguities resulting from aliasing $\Rightarrow$ sampling rate needs to be sufficiently high

$$F_s > 2F_{max}$$

F_{max} is the largest frequency component in the analog signal.

- If the highest frequency contained in the analog signal $x_a(t)$ is $F_{max} = B$ and signal is sampled at a rate

$$\mathbf{F_s > 2F_{max}}$$

Then $x_a(t)$ can be exactly recovered from its sample values

The Sampling Rate $F_N = 2B = F_{max}$ is called Nyquist Rate.

Step 2 : Quantization :

Conversion of a discrete-time continuous valued signal into a discrete-time, discrete-valued (digital) signal by expressing each sample value as a finite number of digits is called quantization.

- It is basically an approximation process.
- Accomplished by rounding or truncating

Quantization Error :Difference between the quantized value and the actual value

$$e_q(n) = x_q(n) - x(n)$$

Where ,

$x_q(n)$ denote sequence of quantized samples at the output of the quantizer

Quantization levels : The values allowed in the digital signal are called quantization levels.

Quantization step size : The distance between two successive quantization levels is called the Quantization step size or resolution. Denoted by Δ.

- The quantizer error $e_q(n)$ is limited to the range

$$-\frac{\Delta}{2} \leq e_q(n) \leq \frac{\Delta}{2}$$

- If x_{max} and x_{min} represents the maximum and minimum values of x(n)
- L is the number of quantization levels

$$\Delta = \frac{xmax - xmin}{L-1}$$

Step 3 : Coding of the quantized samples

- The coding process in A/D converter assign a unique binary number to each quantization level.
- In this process, each discrete value $x_q(n)$ is represented by b-bit binary sequence.
- For L number of quantization levels we need L different binary numbers.

- With a word length of b bits $\longrightarrow$ 2^b different binary numbers
- Hence

$$2^b \geq L \quad \text{or} \quad b \geq \log_2 L$$

Applications :

- **Communication systems:** modulation/demodulation, channel equalization, echo cancellation
- **Consumer electronics:** perceptual coding of audio and video on DVDs, speech synthesis, speech recognition
- **Music:** synthetic instruments, audio effects, noise reduction
- **Medical diagnostics:** magnetic-resonance and ultrasonic imaging, computer tomography, ECG, EEG, MEG, AED, audiology
- **Geophysics:** seismology, oil exploration
- **Astronomy:** VLBI, speckle interferometry
- **Experimental physics**: sensor-data evaluation
- **Aviation:** radar, radio navigation
- **Security:** steganography, digital watermarking, biometric identification, surveillance systems, signals intelligence, electronic warfare
- **Engineering:** control systems, feature extraction for pattern recognition