Data Sampling & Nyquist Theorem

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Signal :

Any physical quantity that varies with time, space, or any other independent variable or variables.

Classification of Signals :

- Continuous -Time Signals
- Discrete -Time Signals
- Continuous -Valued Signals
- Discrete- Valued Signals

Continuous -Time Signals :

- *defined for every value of time*
- *take on values in continuous interval (a , b),where a can be -∞ and b can be ∞.*
- *can be described by functions of a continuous variables*

Discrete -Time Signals :

- *defined only at certain specific values of time*
- *time instants need not be equidistant, but in practice they are usually taken at equally spaced intervals*

The values of a continuous-time or discrete-time signal can be continuous or discrete.

Continuous-Valued Signals :

If a signal takes on all possible values on a finite or an infinite range it is said to be continuous-valued signals.

Discrete-Valued Signals :

If a signal takes on values from a finite set of possible values ,it is said to be a discrete-valued signals.

Signal Processing :

It is an area that deals with operations on or analysis of signals, in either discrete or continuous time. Signals of interest can include sounds, images, time-varying measurement values and sensor data.

Processing of signals includes the following operations :

✓Filtering

✓Smoothing

✓Modulation

✓Digitization

✓A variety of other operations

Categories of signal processing :

Analog Signal Processing: *Most signals of practical interest ,such as speech,biological signals,seismic signals,radar signals and various communication signals such as video and audio signals ,are analog.*
Such signals may be processed directly by appropriate analog systems(such as filters) for the purpose of changing their characteristics or extracting the desired information.
In such case signal has been processed directly in its analog form.

Digital Signal Processing:
In this an analog signal is first converted into the digital signal and then processed to extract the desired information.

Advantages of Digital over Analog Signal Processing

- Digital system can be simply reprogrammed for other applications/ported to different hardware / duplicated
- Reconfiguring analog system means hardware redesign, testing, verification

- DSP provides better control of accuracy requirements
- Analog system depends on strict components tolerance, response may drift with temperature

- Digital signals can be easily stored without deterioration
- Analog signals are not easily transportable and often can't be processed off-line

- More sophisticated signal processing algorithms can be implemented
- Difficult to perform precise mathematical operations in analog form

Analog-to Digital Conversion :

- Most signal of practical interest, such as
  - ✓speech
  - ✓biological signals
  - ✓seismic signals
  - ✓radar signals & sonar signals
  - ✓various communication signals are analog

- To process analog signals by digital means ⟶ Conversion from analog into digital form

⬇

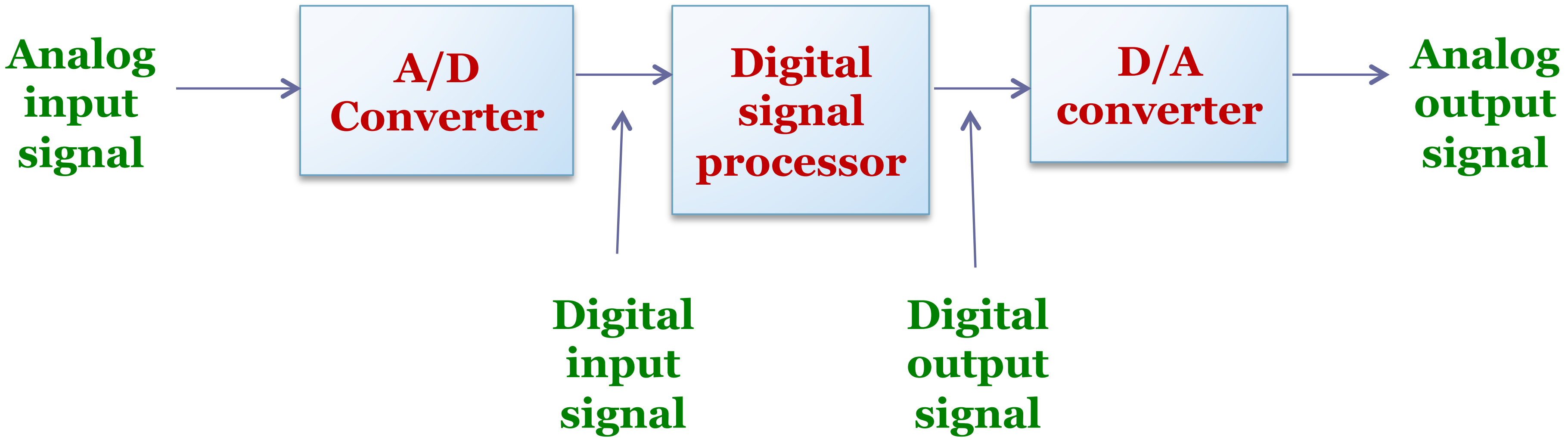
Analog-to-Digital (A/D) conversion

Digital Signal Processing :

For a signal to be processed digitally,

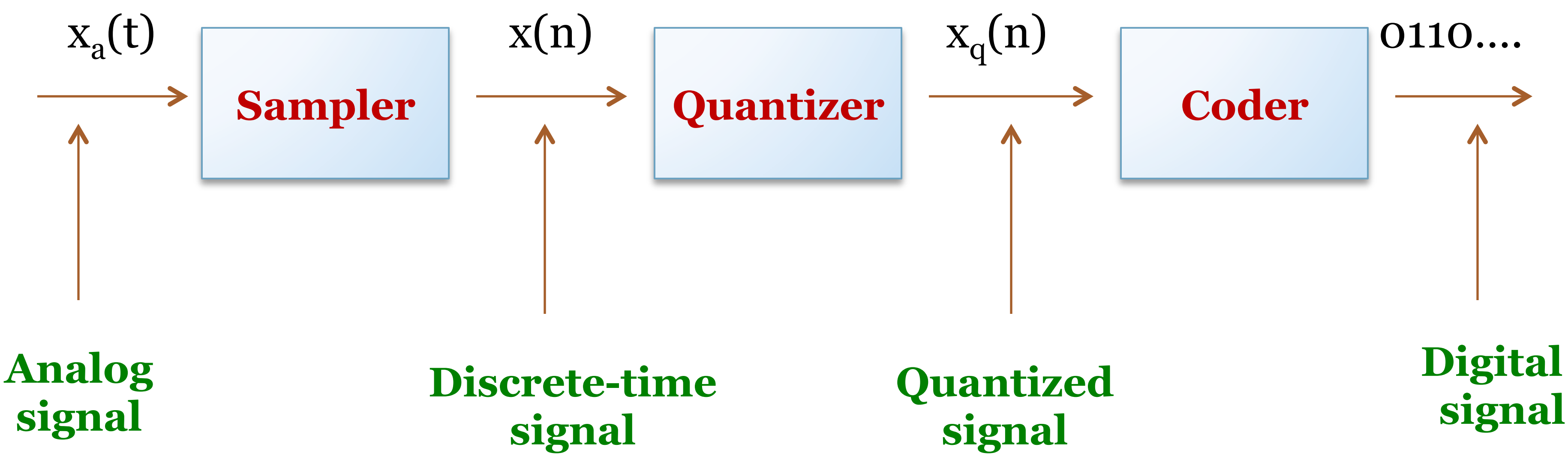
- it must be discrete in time
- Its values must be discrete

Block diagram of a digital signal processing system



A/D conversion is a three-step process :

Step 1 : ***Sampling***
Step 2: ***Quantization***
Step 3: ***Coding***



Step 1 : Sampling of Analog signal

Conversion of continuous-time signal into a discrete-time signal by taking samples of continuous-time signal at discrete time instants.

A continuous time sinusoidal signal is :

$$x_a(t) = A\cos(\Omega t + \theta) \quad , \quad -\infty < t < \infty \qquad (1)$$

Where,

$x_a(t)$: an analog signal

A : is amplitude of the sinusoid

Ω :is frequency in radians per seconds(rad/s)

θ : is the phase in radians

$$\Omega = 2\pi F$$

A discrete-time sinusoidal signal obtained by taking samples of the analog signal $x_a(t)$ every T seconds may be expressed as

$$x(n) = x_a(nT) = A\cos(\omega n + \theta) \quad , \quad -\infty < n < \infty \qquad (2)$$

Where,

n : an integer variable, called sample number

A : is amplitude of the sinusoid

ω : frequency radians per sample

θ : is the phase in radians

$$\omega = 2\pi f$$

f : frequency cycles per samples

T is the **sampling interval** or **sampling period**

$F_s = 1/T$ is called the **sampling rate** or the **sampling frequency**
Hertz)

- **Relationship b/w frequency of analog and digital signal is**

$$f = F/F_s$$

- **Range of frequency variables**

$$-\infty < F < \infty$$

$$-1/2 < f < 1/2$$

- Frequency of the continuous-time sinusoid when sampled at rate F_s must fall in the range

$$-F_s/2 \leq F \leq F_s/2$$

- The highest frequency in the discrete signal is $f = 1/2$,
- With a sampling rate F_s , the corresponding highest value of F is

$$F_{max} = F_s/2$$

Sampling introduces an ambiguity

Limitations of DSP – Aliasing

Most signals are analog in nature, and have to be sampled

→ **loss of information**

➢we only take samples of the signals at intervals and don't know what happens in between

→ **aliasing**

cannot distinguish between higher and lower frequencies

Sampling theorem: to avoid aliasing, sampling rate must be at least twice the maximum frequency component (`bandwidth') of the signal

<u>Sampling Theorem :</u>

- To avoid ambiguities resulting from aliasing ⇒ sampling rate needs to be sufficiently high

$$F_s > 2F_{max}$$

F_{max} is the largest frequency component in the analog signal.

- *If the highest frequency contained in the analog signal* $x_a(t)$ *is* $F_{max} = B$ *and signal is sampled at a rate*

$$\mathbf{F_s > 2F_{max}}$$

Then $x_a(t)$ *can be exactly recovered from its sample values*

The sampling rate $F_N = 2B = F_{max}$ is called Nyquist Rate

Step 2 : Quantization

Conversion of a discrete-time continuous valued signal into a discrete-time, discrete-valued (digital) signal by expressing each sample value as a finite number of digits is called quantization.

- ***It is basically an approximation process.***
- ***Accomplished by rounding or truncating***

Quantization Error :Difference between the quantized value and the actual value

$$\mathbf{e_q(n) = x_q(n) - x(n)}$$

Where ,

$x_q(n)$ denote sequence of quantized samples at the output of the quantizer

- **Quantization levels** : ***The values allowed in the digital signal are called quantization levels.***

- **Quantization step size** : ***The distance between two successive quantization levels is called the Quantization step size or resolution.***
  Dented by Δ.

- *The quantizer error $e_q(n)$ is limited to the range*

$$-\Delta/2 \le e_q(n) \le \Delta/2$$

- *If x_{max} and x_{min} represents the maximum and minimum values of $x(n)$*

- *L is the number of quantization levels*

$$\Delta = xmax - xmin\ / L - 1$$

Step 3 : Coding of the quantized samples

- *The coding process in A/D converter assign a unique binary number to each quantization level.*
- *In this process, each discrete value $x_q(n)$ is represented by b-bit binary sequence.*
- *For L number of quantization levels we need L different binary numbers.*
- *With a word length of b bits* ➡ *2^b different binary numbers*
- *Hence*

$$2^b \geq L \quad \text{or} \quad b \geq \log_2 L$$

Applications :

- **communication systems**

modulation/demodulation, channel equalization, echo cancellation

- **consumer electronics**

perceptual coding of audio and video on DVDs, speech synthesis, speech recognition

- **music**

synthetic instruments, audio effects, noise reduction

- **medical diagnostics**

magnetic-resonance and ultrasonic imaging, computer tomography, ECG, EEG, MEG, AED, audiology

- **geophysics**

seismology, oil exploration

- **astronomy**

VLBI, speckle interferometry

- **experimental physics**

sensor-data evaluation

- **aviation**

radar, radio navigation

- **security**

steganography, digital watermarking, biometric identification, surveillance systems, signals intelligence, electronic warfare

- **engineering**

control systems, feature extraction for pattern recognition

Thank You