The Nonlocal Pancharatnam Phase in Two-Photon Interferometry

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Abstract

The Pancharatnam phase was discovered in the context of amplitude interferometry of polarised light and anticipates Berry’s discovery of the geometric phase. We propose a polarised intensity interferometry experiment which measures the nonlocal Pancharatnam phase acquired by a pair of Hanbury Brown-Twiss photons. The experimental setup involves two polarised thermal sources illuminating two polarised detectors. Varying the relative polarisation angle of the detectors introduces a geometric phase equal to half the solid angle traced out on the Poincaré sphere by a pair of photons. Local measurements at either detector do not reveal the effects of the phase, which appears only in the coincidence counts of the two detectors and is a genuinely multiparticle and nonlocal effect. The phase is an optical analog of the multiparticle Aharonov-Bohm effect which has been measured in Quantum Hall systems.

Keywords : two particle interferometry, geometric phase

1 Introduction

The familiar two slit experiment in quantum mechanics describes the interference of a single particle with itself. However, there are also quantum processes that describe the interference of a pair of particles with itself. As shown by Hanbury Brown and Twiss (\textit{HB-T}) \cite{1} about fifty years ago, such interference is observed in the coincidence counts of photons. Their original motivation was to measure the diameters of stars replacing Michelson interferometry by intensity

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interferometry. Their work was initially met with scepticism because the quantum mechanical interpretation of the proposed experiment was unclear at the time. The resulting controversy led to the birth of the new field of quantum optics. Intensity interferometry is now routinely used in a variety of fields, from nuclear physics [2, 3] to condensed matter [4].

In the nineteen eighties, Berry discovered [5] the geometric phase in quantum mechanics, which has now been applied and studied in various contexts [6]. It was soon realised that Berry’s work was anticipated by Pancharatnam’s work [7] on the interference of polarised light, Pancharatnam’s work is now widely recognised as an early precursor of the geometric phase [8], with a perspective that is far more general [9] than the context in which it was discovered by Berry.

Buttiker [10] noted in the context of electronic charge transport that two-particle correlations can be sensitive to a magnetic flux even if the single particle observables are flux insensitive. The effect of the flux is visible only in current cross correlations and is a genuinely nonlocal and multiparticle Aharonov-Bohm effect [11]. This effect has been experimentally seen in intensity interferometry experiments carried out using edge currents in quantum Hall systems [4] and the theory was further developed in [12, 13] and the possibility of controlled orbital entanglement and the connection to Bell inequalities mentioned.

In this paper, we propose a new experiment with polarised light, which shows geometric phase effects only in the intensity correlations $G^2$ and not in the lower order correlations $G^1$. The two photon Pancharatnam phase effect is also nonlocal in the precise sense that it cannot be seen by local measurements at either detector. Coincidence detection of photons in two detectors yields counts which are modulated by a phase which has a geometric component as well as the expected dynamical (or propagation) phase. Unlike in earlier studies [14, 15], the effects of the geometric phase are seen only in the cross correlation counts of two detectors. Neither the count rate nor self correlation of the individual detectors shows any such geometric phase effects. The phase is given by half the solid angle enclosed on the Poincaré sphere by the total circuit of a pair of HB-T photons and as expected, is achromatic.

The experimental setup is described in the next section (Sec. 2) and a theoretical analysis is given in Sec. 3. We conclude with a discussion and a comparison with previous work in Sec. 4.

## 2 Proposed Experiment

The experiment consists of having two thermal sources $S_1$ and $S_2$ illuminate two detectors $D_3$ and $D_4$ (Fig. 1). This setup is very similar to the HB-T experiment [1]. The only difference is in the use of polarisers (shown in red online), which select a particular state of polarisation. The source $S_1$ is covered by a polaroid $P_R$, which only permits Right Hand Circular light to pass through it, while the source $S_2$ is covered by a polaroid $P_L$, which only permits Left Hand Circular light to pass through. The light is incident on detectors $D_3$ and $D_4$ after passing through polaroids $P_3$ and $P_4$ respectively that only permit linearly polarised light to pass through. The angle $\varphi_{34}$ between the axes of $P_3$ and $P_4$ and the detector separation $d_D$ can be continuously varied in the experiment. The measured quantity is the coincidence count $C$ of photons received at detectors
Figure 1: Schematic diagram of the proposed experiment: $S_1$ and $S_2$ are thermal sources, covered by polaroids which only pass right and left circular light respectively. The two detectors $D_3$ and $D_4$ receive only linear polarizations. The angle $\varphi_{34}$ between the axes of the linear polarizers can be continuously varied. The dashed and solid lines represent photons from the two sources $S_1$ and $S_2$ respectively.

$D_3$ and $D_4$,  

$$C = \mathcal{G}^2 = \frac{\langle N_3 N_4 \rangle}{\langle N_3 \rangle \langle N_4 \rangle},$$  

(1)

where $N_3$ and $N_4$ are the photon numbers detected at $D_3$ and $D_4$ per unit time (per unit bandwidth).

As in the HB-T interferometer, we would expect the coincidence counts to vary with the propagation phases and so the counts would depend on the detector separation $d_D$ and the wavelength $\lambda$ of the light. The new effect that is present in the polarised version is that we expect the coincidence counts to also depend on $\varphi_{34}$ the angle between the linear polarisers $P_3$ and $P_4$ and to be modulated by a geometric phase of half the solid angle on the Poincaré sphere shown in Fig. 2.

The geometric phase is achromatic, unlike the propagation phases mentioned above. Note that the path traversed on the Poincaré sphere is not traced by a single photon, but by a pair of HBT photons. Thus the experiment explores a new avatar of the geometric phase in the context of intensity interferometry.

3 Theory

In this section we calculate the expected coincidence counts for the detectors $D_3$ and $D_4$ and show that these counts depend on the geometric phase. For ease of calculation we suppose that we are dealing with a single frequency i.e. a monochromatic beam. In fact the detectors will have a finite acceptance bandwidth, which has to be incorporated in a more realistic calculation. The principle of the effect comes across better in the present idealised situation.

We write $a_1^\dagger$ and $a_2^\dagger$ for the destruction operators of the photon modes at the sources $S_1$ and $S_2$ where $\alpha$ runs over the two polarisation states. The modes just after the polaroids $P_R$ and
Figure 2: The path on the Poincaré sphere that determines the geometric phase. The angle $\varphi_{34}$ between the linear polaroids determines the width of the lune on the Poincaré sphere and the geometric phase.

$P_L$ are represented by projections $a_\alpha^L = P_{\alpha\beta} a_\beta^L$ and $a_\alpha^R = P_{\alpha\beta} a_\beta^R$ where a sum over repeated Greek indices is understood and the projection matrices $P_R$ and $P_L$ onto the right and left circular states represent the action of the polaroids. The modes at the detectors are characterised by the destruction operators $a_\alpha^3$ and $a_\alpha^4$. We suppose that the separation $l$ between the sources and the detectors is much larger than the separation between $d_S$ between the sources and the separation $d_D$ between the detectors i.e., $l >> d_S, d_D$. When light is emitted by a source and received by a detector, it suffers propagation phases and decrease of its amplitude inversely with distance. These effects are captured in the functions $u_{ij} = \frac{1}{l} \exp \{ i[k(|\vec{r}_i| - |\vec{r}_j|) - \omega t] \}$, where $\omega$ is the frequency of the light, $k$ is the wave vector and $\vec{r}_i$ and $\vec{r}_j$ the locations of the detector and source. With this notation, we express $a_\alpha^3$ as

$$a_\alpha^3 = P_{3\beta} \left[ P_{L}^{\beta\gamma} a_{\gamma}^2 u_{32} + P_{R}^{\beta\gamma} a_{\gamma}^1 u_{31} \right],$$

(2)

and its Hermitean conjugate $a_\alpha^3$ as

$$a_\alpha^3 = \left[ \bar{u}_{32} a_\gamma^2 P_{L}^{\gamma\beta} + \bar{u}_{31} a_\gamma^1 P_{R}^{\gamma\beta} \right] P_{3}^{\alpha\beta},$$

(3)

where the overbar stands for complex conjugation and we use the fact that the $2 \times 2$ Hermitean projection matrices $P$ satisfy $P^2 = P$ and $P_{\alpha\beta} = P_{\beta\alpha}$.

The quantities of interest $^1$ are $\langle N_3 \rangle$, $\langle N_4 \rangle$, being the photon counts per unit time (per unit bandwidth) at the two detectors ($D_3$ and $D_4$) and $\langle : N_3 N_4 : \rangle$, the coincidence counts, where the $::$ stands for normal ordering. $N_3$ is given by $(a_\alpha^3 a_\beta^3)$ and $N_4$ by $(a_\alpha^4 a_\beta^4)$. We easily find

$$N_3 = [ \bar{u}_{32} a_2^\alpha (P_L P_3)^{\alpha\beta} + \bar{u}_{31} a_1^\alpha (P_R P_3)^{\alpha\beta} ][ P_{L}^{\beta\gamma} a_{\gamma}^2 u_{32} + P_{R}^{\beta\gamma} a_{\gamma}^1 u_{31} ],$$

$$N_4 = [ \bar{u}_{42} a_2^\alpha (P_L P_4)^{\alpha\beta} + \bar{u}_{41} a_1^\alpha (P_R P_4)^{\alpha\beta} ][ P_{L}^{\beta\gamma} a_{\gamma}^2 u_{42} + P_{R}^{\beta\gamma} a_{\gamma}^1 u_{41} ].$$

(4)

$^1$For any general operator $\hat{O}$, $\langle \hat{O} \rangle = \text{Tr}[\hat{O} \hat{\varrho}]$ where $\varrho$ is the normalised thermal density matrix $\varrho = \exp{-\beta H}/Z$ with $Z = \text{Tr}[\exp{-\beta H}]$ and $H = (a^\dagger a + 1/2)\omega$. 

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The experimentally measured quantity is
\[ C = \frac{\langle :N_3 N_4 : \rangle}{\langle N_3 \rangle \langle N_4 \rangle}. \] (5)

We readily find
\[ \langle N_3 \rangle = \bar{u}_{31} u_{31} (P_R P_3 P_R P_3)^{\alpha \beta} \langle a_1^{\alpha} a_1^{\beta} \rangle + \bar{u}_{32} u_{32} (P_L P_3 P_L P_3)^{\alpha \beta} \langle a_2^{\alpha} a_2^{\beta} \rangle, \]
\[ \langle N_4 \rangle = \bar{u}_{41} u_{41} (P_R P_4 P_R P_4)^{\alpha \beta} \langle a_1^{\alpha} a_1^{\beta} \rangle + \bar{u}_{42} u_{42} (P_L P_4 P_L P_4)^{\alpha \beta} \langle a_2^{\alpha} a_2^{\beta} \rangle. \] (6)

From the thermal nature of the sources, \( \langle a_1^{\alpha} a_1^{\beta} \rangle = \langle a_2^{\alpha} a_2^{\beta} \rangle = \delta^{\alpha \beta} n_B \) where \( n_B \) is the Bose function \( (\epsilon^{\beta \omega} - 1)^{-1} \) and \( \beta \) the inverse temperature. And we arrive at
\[ \langle N_3 \rangle = \langle N_4 \rangle = \frac{n_B}{l^2}. \] (7)

The computation of \( \langle :N_3 N_4 : \rangle \) is slightly more involved but straightforward. The product \( N_3 N_4 \) (see Eq. (4)) is a product of four brackets each of which has two terms. When the four brackets are expanded, there are sixteen terms, of which ten vanish. The six nonzero terms combine to give
\[ \langle :N_3 N_4 : \rangle = n_B^2 \left[ \frac{3}{2 l^4} + \bar{u}_{32} u_{31} \bar{u}_{41} u_{42} \text{Tr} [P_L P_3 P_R P_4 P_L P_4] + \bar{u}_{31} u_{32} \bar{u}_{42} u_{41} \text{Tr} [P_R P_3 P_L P_4 P_R] \right], \] (8)

Only the second and third terms in Eq. (8) contain the propagation and geometric phases. The sequence of projections can be viewed as a series of closed loop quantum collapses [9] given by
\[ \text{Tr} [P_R P_3 P_L P_4 P_R] = \langle R | 3 \rangle \langle 3 | L \rangle \langle L | 4 \rangle \langle 4 | R \rangle = \frac{1}{4} \exp \left\{ \frac{i \Omega}{2} \right\}, \] (9)

where \( \Omega \) is the solid angle subtended by the geodesic path \( | R \rangle \rightarrow | 3 \rangle \rightarrow | L \rangle \rightarrow | 4 \rangle \rightarrow | R \rangle \) at the center of the Poincaré sphere (Fig. 2). Apart from the phase, the projections also result in an amplitude factor of \( 1/4 \) [7] since projections are non-unitary operations leading to a loss in intensity \(^2\). Using Eqs. (7) and (8), we can compute the correlation in Eq. (5). The final theoretical expression for \( C \) is
\[ C = \frac{3}{2} + \frac{1}{2} \cos \left[ k(r_{31} + r_{42} - r_{32} - r_{41}) + \frac{\Omega}{2} \right], \] (10)

where \( r_{ij} \) is the distance between \( j \)th source (\( j = 1 \) or 2) and \( i \)th detector (\( i = 3 \) or 4). In the limit \( l \gg d_s, d_D \), we can express
\[ C = \frac{3}{2} + \frac{1}{2} \cos \left[ \vec{d}_S \cdot (\vec{k}_4 - \vec{k}_3) + \frac{\Omega}{2} \right], \] (11)

where \( \vec{k}_i = k \hat{r}_i \) is the wavevector of light seen in the \( i \)th detector.

\(^2\)The value of 1/4 is due to our special choice of the polaroid states.
It is also easily seen that the self correlation $\langle N_3 N_3 : \rangle$ ($\langle N_4 N_4 : \rangle$) can be obtained by replacing 4 by 3 (3 by 4) in Eq. (8) above. In this case, that the sequence of projections $\text{Tr} \{ P_R P_3 P_L P_3 P_R \}$ ($\text{Tr} \{ P_R P_4 P_L P_4 P_R \}$) subtends a zero solid angle and the geometric contribution to the phase vanishes. Thus neither the photon counts $\langle N_3 \rangle$, $\langle N_4 \rangle$ (see Eq. (7)) in individual detectors nor the self correlations $\langle N_3 N_3 : \rangle$, $\langle N_4 N_4 : \rangle$ reveal the geometric phase.

$C$ depends on the experimentally tunable parameters $d_D$ and $\varphi_{34}$. The geometric part is achromatic and depends linearly only on $\varphi_{34}$. The propagation part in the phase carries the dependence on $d_D$ as well as on the energy. By changing the angle $\varphi_{34}$ between the axes of the two polaroids, we can conveniently modulate the geometric component $\Omega$. If the propagation and geometric phases are set to zero, we find that the correlation $C$ takes the value 2, just as in original HB-T interferometry.

4 Conclusion

We have proposed a simple generalisation of the HB-T experiment which uses the vector nature of light to produce a geometric phase. The only difference between the proposed experiment and the HB-T experiment is the presence of polarisers at the sources and detectors. These polarisers cause a geometric phase to appear in the coincidence counts of two detectors which receive linearly polarised light. Neither the count rates nor the self correlations of individual detectors show any geometric phase effects. These appear solely in the cross correlations in the count rates of the detectors. This is a new result of a conceptual nature, which may not have been guessed without our present understanding of the Pancharatnam phase. While the experiment we propose is no harder than the original HB-T experiment, it has not so far been performed. While there is every reason to expect that the outcome will be as predicted by the theory, it would be a good demonstration of a purely multiparticle and nonlocal geometric phase in optics. We hope to interest experimentalists in this endeavour.

The experiment we propose with photons is considerably simpler to perform than the related experiment [4] which has been done for electrons in the quantum Hall effect. In the experiment of Ref.[4], intensity interferometry has been done using a pair of electrons which together enclose an Aharonov-Bohm [11] flux due to an applied magnetic field. They use an Aharonov-Bohm flux to modulate the geometrical component of the two particle cross correlation via coupling to the orbital degree of freedom of electrons while the spin remains frozen. Just as in our proposed experiment, the effects of the flux are absent in the individual currents and in their self correlations. It is only the cross correlation between electron currents that reveals the dependence on the Aharonov-Bohm flux. In our case, the geometric phase cannot be induced by an Aharonov-Bohm magnetic flux since photons are neutral particles. We use the polarisation (spin) of photons to produce a geometric phase in the two particle cross correlation. Our proposed experiment differs from [4] in that the geometric phase arises purely [16] due to projection effects: the polarisers reject photons of a certain polarisation and this leads to a Pancharatnam phase, in a manner well known in amplitude interferometry. There are also sign differences owing to the different statistics of the particles involved. The main point of the present paper is that a small modification of HB-T intensity interferometry experiment leads to an optical analogue of the multiparticle
Aharonov-Bohm effect. There arises the possibility remarked already in [12, 13] of tuning the orbital entanglement of photons by using the geometric phase.

There have been studies [14, 15], which study the geometric phase in intensity interferometry. However, in these experiments (as we noted above) the geometric phase is also manifest in the lower order correlations and local effects at each detector. These may be viewed as two copies of the single photon geometric phase, which either add or cancel (depending on their relative sign) rather than a genuine two particle effect. As a result the phase observed in these experiments is not half the solid angle on the Poincaré sphere but twice this value (or zero). There is also a difference in the nature of the source. The experiment described in [14, 15] uses a pair of entangled photons as a source, while our sources are incoherent and thermal just like the original HB-T experiment. It is elementary to check that if our sources were coherent laser beams, the effect in question would disappear. This too is exactly in accord with the HB-T interferometer.

In astronomy, the HB-T interferometer has been largely displaced by the older Michelson Technique. However, in recent times, there have been proposals [17, 18] to revive HB-T interferometry. Indeed the proposals involve not just two photon processes, but three and four photon correlations. The achromatic nature of the Pancharatnam phase in intensity interferometry could prove useful in these applications, analogous to suggestions [19] made in the context of Michelson interferometry.

We have restricted ourselves to the simplest situation of two sources and two detectors each covered by polaroids. Going to more sources and more detectors is simple in principle. One could consider say a “three slit experiment” in which there are three slits illuminated by a background thermal source through which light falls on three detectors, which are covered by polaroids, each of which filters out all but one state on the Poincaré sphere. One would find then that even within ordinary quantum mechanics, there are “three slit effects” of the geometric phase variety, which are not revealed in any “two slit” interference between pairs. This assumes significance in the light of theoretical proposals [20] and experimental efforts [21] to detect three slit effects that go beyond quantum mechanics. Since these experiments [21] are null experiments, we have to be careful to rule out all possible three slit effects that are present in quantum mechanics. Geometric phase effects which involve three photons are an example of such three slit effects.

Finally, the ideas here can be extended to other particles than photons. The principle of the experiment is the same, but for Fermions, the calculation leads to a few extra minus signs because of the commutators being replaced by anticommutators. The experiments of [4] could be performed with spin polarised sources and detectors, obviating the need for an external Aharonov-Bohm flux. Similarly, neutrinos obey Fermi statistics and can be expected to show anti-coincidence rather than coincidence in their cross correlations. For experimental convenience, the spin degree of freedom can be replaced by the flavour degree of freedom. As explained in [22, 23] the passage of neutrinos through matter and vacuum leads to analogue birefringent effects. Given the interest in interferometry, in astronomy, particle physics, optics, atomic physics and condensed matter physics, we expect that our proposal would lead to conceptual and technical advances in all these areas.

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References


