Reply to the comment on "Topological phase in two flavor neutrino oscillations"

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Abstract

In a recent paper, we showed that there is a neat geometric interpretation of two flavor neutrino oscillation formulae, and that the geometric phase involved in the physics of oscillations is restricted to be topological as long as CP is conserved. This paper has been criticised by Bhandari. In the present note, we show that the criticisms are not valid and only reflect his failure to understand some crucial points.

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In Ref. [1], we have shown that the coherent phase involved in the neutrino oscillation formalism can be decomposed into two parts, one corresponding to the usual dynamical phase and the other part being geometric. This study was carried out for the specific case of CP conserving two flavor neutrinos. In this context we noted that the geometric phase is actually topological [1].

The author of the comment [2] claims to give a simple derivation of the main result obtained in [1] and then goes on to make some strong, unjustified and inappropriate remarks towards the end. We disagree with all of what is claimed in [2] and in what follows, we rebutt these claims.

First let us address the scientific content of the comment [2]. The author of [2] repeats the derivation of the two flavor neutrino oscillation formulae as presented in our paper [1]. There is no disagreement till Eq. (4) in [2]. In Ref. [1], we note that the individual interference terms (with the dynamical phase removed) appearing in the expressions for survival ($c_+^\star c_- A_\alpha$ in the notation of [2]) and appearance ($c_+^\star c_- A_\beta$ in the notation of [2]) probabilities can be written as $re^{i ß}$ where ß is the Pancharatnam phase. We find that this phase can only take values 0 or π for CP conserving interactions.

The disagreement appears because the author of [2] is computing the relative phase between the interference terms of survival (Eq. (3)) and appearance (Eq. (4)) probabilities i.e. $A_\beta^\star A_\alpha$ (see Eq. (6) in [2]) which *differs* from our quantity of interest. It is therefore no surprise that the author of [2] gets a different answer: his phase is always $\pm\pi$. This result follows easily from unitarity, but is *irrelevant to our discussion.* His final expression Eq. (6) (in [2]) only contains *four* states ($|\,\nu_\alpha\,\rangle, |\,\nu_\beta\,\rangle, |\,\theta_2, +\,\rangle, |\,\theta_2, -\,\rangle$) and not six ($|\,\nu_\alpha\,\rangle, |\,\nu_\beta\,\rangle, |\,\theta_1, +\,\rangle, |\,\theta_1, -\,\rangle, |\,\theta_2, +\,\rangle, |\,\theta_2, -\,\rangle$) for the general case of varying density matter (see Eq. (16) of [1]).

On page 3, the author of [2] also gives a connection of the $\pm\pi$ phase in neutrino system with phase shifts observed in polarisation optics. This correspondence is discussed in detail in [1]. This analogy is precisely the reason for viewing neutrino oscillations using ideas current in polarisation optics. We would like to add that such analogous optics experiments to demonstrate the physics involved in two flavor neutrino oscillations have been carried out recently by Weinheimer [4]. However spatial split beam experiments which are common in optics are impossible for neutrinos because of their low refractive index. To get around this, we consider neutrino oscillations in terms of a split-beam two-path interference experiment in *energy space.* The author of [2] missed this most crucial point of [1]. From this viewpoint, the cross term in the survival and appearance probability can be viewed as a series of quantum collapses with intermediate adiabatic evolutions (in varying density matter) which may or may not enclose a solid angle in the ray space. The phase of the cross terms (with the dynamical phase removed) is the Pancharatnam phase. Hence in [1], we are only interested in the individual cross terms appearing in the probability and not in the relative phase as computed in [2]. This settles the major scientific objection raised in [2].

Next, we address the criticism on page 3 of [2] regarding misleading statements made in

[1]By our definition, the topological phase refers to phase factors that are insensitive to small changes in the circuit (and are invariant under deformations of circuit), while geometric phases are sensitive to such changes.

our paper [1]. Before we predict the value for the geometric phase [2] using Pancharatnam's ideas [3], we have very clearly used the phrase *"Upon removing the dynamical phase"* on page 7 (after Eq. (16) in [1]) which the author of [2] missed and therefore was led to an incorrect understanding of the paper. Hence our claim is correct. We would like to stress that *the dynamical phase is by no means ignored in the paper [1]* - it is present in all expressions including the final equations (Eq. (21) of [1]) when we compare our result in the special case with standard expressions for probability used in literature. The author of [2] claimed (wrongly of course) that geometric phase factors in the cross terms in Eqs. (3) and (4) are constant in time only for the case of vacuum. Whereas, what we claimed was that the geometric part in the total phase is always topological i.e. restricted to take values 0 or π irrespective of the properties of medium as long as we have a CP conserving form of the total Hamiltonian which means it holds for vacuum or any medium with either a constant or slowly varying density. This is because for the CP conserving case, the states are real and the cyclic loop formed by collapses of neutrino states is always restricted to a great circle on the Poincaré sphere. The topological phase depends on the actual winding around this equatorial great circle and is either 0 or π. Our result of "π anholonomy" is also consistent with the first available papers (around 1958) on geometric phases in CP conserving two by two case in the field of molecular physics [5].

Now we discuss the objection raised on page 3 in [2] regarding a sentence in the abstract :

"Our study shows for the first time that there is a geometric interpretation of the neutrino oscillation formulae for the detection probability of neutrino species."

We would like to mention that the full sentence in the abstract of [1] is actually the following (and not what is said in the comment [2])

"For the minimal case of two flavors and CP conservation, our study shows for the first time that there is a geometric interpretation of the neutrino oscillation formulae for the detection probability of neutrino species."

We think this statement is apt because of the reasons that have already been listed in detail in the paper [1]. We repeat them here. Note that the statement is made for the minimal case of two flavors and CP conservation. The first point to note is that in two flavor case, we can have up to three independent parameters appearing in the Hamiltonian. In case of vacuum or constant density matter, there is no varying parameter while for varying density matter, there is only one such parameter (the electron number density). Hence appearance of cyclic Berry phase is ruled out. Of course one can think of the generalized geometric phase and some people claimed appearance of geometric phase in the two flavor case, but it should be noted that such terms appeared at the level of amplitudes and not probabilities. In [1], we clarified that due to the inherent limitation of designing a split-beam experiment with neutrinos in the physical space, it was impossible to access phases appearing at the level of amplitudes. So the existing literature led to the belief that the formulae for two flavor neutrino oscillation probability were devoid of any geometric phase contribution. However, contrary to all the existing claims, our study showed that there exists a topological contribution to the total phase at the probability level even for the

[2]The "geometric phase" is by definition the phase that is not dynamical and it is obtained after dropping the dynamical phase from the interference term.

minimal case of two flavors and CP conservation. And, to the best of our knowledge, this was noted for the first time in [1].

Regarding the correctness of the following statement as questioned on page 3 in [2] :

"More precisely the standard result for neutrino oscillations is in fact a realization of the Pancharatnam topological phase."

Because of the topological phase being inherently built into the structure of the neutrino mixing matrix, it is true that the standard formalism of oscillations is actually a realization of Pancharatnam's phase. The minus sign appearing in the two by two orthogonal rotation matrix has the interpretation of being the Pancharatnam phase of π.

There were some general comments in [2] concerning relevance and usefulness of our paper to which we would like to respond. There was some confusion among the neutrino community as to whether the geometric phase is an extra contribution to be added to the standard treatment. Our paper shows that the geometric phase is topological (for CP conserving, two flavor case) and contained in the standard treatment of neutrino oscillations. It therefore serves a useful scientific purpose.

To counter the criticism in [2] regarding use of "high sounding language" in [1] (Physical Review), we would like to remark that the language used is quite accessible to the readership of Physical Review and considerably simpler than for example, that of the Letter [7], which is far more technical and involves abstract differential geometry. This criticism [2] sounds very strange coming from a co-author on the PRL paper!

Having thus disposed of the rather meagre scientific content of the objections raised in [2], we now come to what appears to be the main point of this manuscript : the question of priority. The main claim made in [2] is that the idea of using the *shortest geodesic rule* was first put forward in [6] and not in [7]. However this can not be sustained and the author has been caught in the unfortunate position of having been "scooped" by himself. The author of [2] objects to the "use of term shorter along with the reference to [7]" in our paper [1] by claiming that the words "shortest" or "shorter" appear nowhere in the formulation of the geodesic rule in the paper by Samuel and Bhandari [7]. This claim is patently false. The word "shortest" appears in footnote 11 on page 2342 of [7] and logically complete supporting argument given there. The footnote is not only a part of the paper but very much a part of the proof of the geodesic rule in question. We first reproduce the footnote from [7] which contains the word shortest (underlined below),

"In fact, $\langle\, \tilde{\phi}(0)\,|\,\tilde{\phi}(s)\,\rangle$ is also positive if $|\,\tilde{\phi}(s)\,\rangle$ is the <u>shortest</u> geodesic connecting $|\,\tilde{\phi}(0)\,\rangle$ with $|\,\tilde{\phi}(1)\,\rangle$. Along this curve, $\langle\, \tilde{\phi}(0)\,|\,\tilde{\phi}(s)\,\rangle$ never vanishes and, since it was positive to start with, remains so."

While the footnote is enough to convincingly dismiss the claim, we would like to add the following. The author of the comment [2] is evidently confused about the use of "any" in the phrase "any geodesic arc" on page 2341 in his own paper [7]. A reading of the relevant paragraph shows that the discussion is carried out in $\mathcal{N}$ the set of normalizable states, which admits gauge freedom. The geodesic equation is written in $\mathcal{N}$ and is gauge invariant. This gauge freedom means that gauge copies of geodesics are also geodesics. "Any" refers to any of the gauge copies. The discussion of the footnote referred to earlier is in the *ray space* and clearly states that it is the *shortest geodesic* that is relevant.

In closing we remark that the tone and language of [2] are inappropriate for any scientific journal. The scientific content of the comment is meagre and the main point of priority unsustainable [8]. We believe that such unseemly and fruitless controversies over minute points of priority [8] are a drain on the time of authors, editors, referees and readers and do not belong in any scientific journal.

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[8] Actually, the distinction between the greater and shorter arc on the Poincaré sphere appears even earlier in papers by Pancharatnam [3], Berry [9] and possibly others!

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