Topological phase in two flavor neutrino oscillations :

A new interpretation from geometry

Poonam Mehta

Raman Research Institute,

Bangalore, India.

Email – poonam@rri.res.in

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Outline of the introduction

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- Anomalies in the neutrino sector

  in a wide variety of systems (sun, atmospheric, reactor, accelerator)

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- In SM, neutrinos are massless $\rightsquigarrow$ can not oscillate

  Neutrino flavor oscillations is the only firm evidence *in favor of* physics beyond the Standard Model of particle physics, even though there are other strong indications (BAU, DM, DE, ..)

Information gleaned from experiments

- Experiments observing neutrinos from

  cosmic ray interactions in the atmosphere, sun, reactor, terrestrial experiments reveal that neutrinos undergo a change of flavor

  - Atmospheric expts. $\boxed{\nu_\mu \longrightarrow \nu_\tau \text{ and } \bar{\nu}_\mu \longrightarrow \bar{\nu}_\tau}$ (SuperK)
  - Solar expts. $\boxed{\nu_e \longrightarrow \nu_\mu \text{ or } \nu_\tau}$ (Homestake, SAGE, GALLIUM, Kamiokande, SuperK, SNO, Borexino)
  - Reactor anti-nu expts. $\boxed{\bar{\nu}_e \longrightarrow \bar{\nu}_x}$ (KamLAND, CHOOZ, LSND, MiniBooNE)
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Future expts. in, R. Gandhi, PM and S. Uma Sankar, INO/HRI/2005/03 **(2005)**

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Different scales of energy E and path lengths L allow for different sensitivities to mass-squared differences in vacuum or in matter

- Very wide range for tunable parameters L, E.

Present data

- 3 flavors $\nu_e, \nu_\mu, \nu_\tau \Rightarrow$ 9 leptonic mass and mixing parameters

  PMNS mixing matrix contains $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_2, \alpha_3$

  $$\mathbb{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times$$
  $$\mathrm{diag}[1, e^{i\alpha_2/2}, e^{i\alpha_3/2}]$$

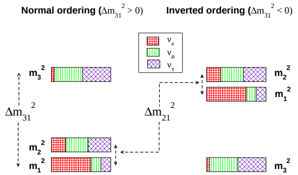
  where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$.

M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep. 460, 1 **(2008)**,
R. Z. Funchal, talk at ν2008

Present data

- Mass and mixing parameters

- Low energy oscillation data -
  - 3 mixing angles :
    $\boxed{\theta_{12} \approx 32^o \text{ , } \theta_{23} \approx 45^o, \ \theta_{13} \lesssim 10^o \text{ (upper bound)}}$
  - 2 mass-squared differences :
    $\boxed{\Delta m^2_{21} \simeq 7.7 \times 10^{-5} eV^2 \text{ and } |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3} eV^2}$
  - $\Delta m^2_{\odot}/\Delta m^2_{atm} \sim 0.03$
  - Solar data : $\Delta m^2_{\odot} > 0$
  - Sign of Δm^2_{31} ?
  - Dirac CP phase δ ?



Present data

- Mass and mixing parameters

- Absolute value of neutrino masses - inaccessible via oscillation
  - β-decay experiments, $m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$, $\boxed{m_\beta < 1.8 \text{ eV}}$ (Mainz+Troitsk)
  - $0\nu\beta\beta$ experiments (sensitive to Majorana nature), $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i| = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{i\alpha_3} m_3|$, $\boxed{m_{\beta\beta} = 0.16 - 0.52 (0 - 0.25) \text{ eV}}$ (Heidelberg-Moscow (Cuoricino)) LNV
  - Cosmology, $\Omega_\nu \propto \sum = \sum_i m_i$, $\boxed{\sum < 1.3 eV}$ (WMAP5)

- Two Majorana phases (α_2, α_3) - inaccessible via oscillation
  $0\nu\beta\beta$ experiments

Why do the neutrinos oscillate ?

- Neutrinos are produced and detected via weak interaction

  Weak (flavor) eigenstates differ from the stationary (mass) states of the Hamiltonian, infact they are linear superpositions of the stationary mass states

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- Incoherent scattering cross-section is usually negligible $\Rightarrow$ sustained coherence over astrophysical length scales

Two flavor approximation

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- Conventionally, atmospheric data can be explained by Δm^2_{32} and θ_{13} and solar data by Δm^2_{21} and θ_{12}.

Dispersion relation for two flavor neutrinos

- In the ultra-relativistic limit, the relativistic dispersion relation

In vacuum, neutrinos obey

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$$

Assume equal and fixed momenta, $p_1 = p_2 = p$ (monochromatic)

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- Two flavor neutrinos $\rightsquigarrow$ Two state quantum system
- Hilbert space of this system can be mapped onto a Blöch sphere (analogous to Poincaré sphere in optics)
- In vacuum, the mass-squared difference and mixing between the two neutrinos leads to flavor oscillations i.e. $\mathbb{H}_{fl} = \mathbb{U}^\dagger(\theta)\mathbb{H}_m\mathbb{U}(\theta)$ is not diagonal.

Inclusion of matter effects (SM interactions)

In ordinary matter

$$\begin{aligned}\mathbb{H}_\nu &= \left(p + \frac{m_1^2+m_2^2}{4p} + \frac{V_C}{2} + V_N\right)\mathbb{I}\\ &+ \frac{1}{2}\begin{pmatrix} V_C - \omega\cos 2\theta & \omega\sin 2\theta \\ \omega\sin 2\theta & -(V_C - \omega\cos 2\theta)\end{pmatrix}\end{aligned}$$

- $V_C = \sqrt{2}G_F n_e$ and $V_N = -\sqrt{2}G_F n_n/2$ are the SM induced potentials due to neutrino matter (e, n, p) interactions and $\omega = \delta m^2/2p$

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Wolfenstein (1978), Mikhevev and Smirnov (1985)

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  Wolfenstein (1978), Mikhevev and Smirnov (1985)
- Absence of FCNC $\rightsquigarrow$ if vacuum mixing and mass-splitting is zero, then matter does not really matter (in oscillations).

Neutrino refraction in vacuum and media

L. Wolfenstein, Phys. Rev. D 17, 2367 (1978), P. Langacker and Liu, Phys. Rev. D 46, 4140 (1992)

- In vacuum, neutrino refraction arises due to the $m^2/2E^2$ term

$$n_{refr} - 1 = \frac{p}{E} - 1 \simeq -\frac{m^2}{2E^2}$$

- In medium, one can use coherent forward scattering amplitudes $f(0)$ to compute the n_{refr},

$$n_{refr} - 1 \quad \simeq \quad -\frac{m^2}{2E^2} + \frac{2\pi}{E^2} n_e f(0)$$

- For $\nu_e - e$ CC scattering $(E's << M_W)$, to leading order in G_F,

$$f(0) \quad = \quad -\frac{E}{2\pi n_e}(V_C)$$

- At zero temperature, $n_{refr} \propto n_e$, neglecting vacuum term,

$$\begin{aligned} n_{refr} - 1 \quad &\simeq \quad -\frac{(\sqrt{2} G_F n_e)}{E} \\ &= \quad -7.6 \times 10^{-14} Y_e \frac{\rho}{[\text{ g cm}^{-3}]} \frac{[\text{ eV}]}{E} \end{aligned}$$

Effects in optics and their counterparts in the neutrino system

Effect of medium can be described in terms of

$$\mathbb{H} = D\mathbb{I} + A\sigma_x + B\sigma_y + C\sigma_z$$

D just gives an overall phase, while A, B, C generate non-trivial optical effects

Optical effects

- Circular Birefringence (Optical activity)
  $C, D \neq 0$ while $A, B = 0$
- Linear Birefringence (Wave plate)
  $A, D \neq 0$ while $B, C = 0$
- Elliptic Birefringence (Quartz plate)
  $A, B, C, D \neq 0$ (most general)
- Dichroism (absorptive effect)
  $\mathbb{H}$ need not be Hermitian

Neutrino oscillations

- Oscillations in vacuum
  $\equiv$ Elliptic birefringence

$$A = \frac{\omega}{2}\sin 2\theta, B = 0, C = -\frac{\omega}{2}\cos 2\theta$$

- Oscillations in normal matter + SM
  $\equiv$ Elliptic birefringence

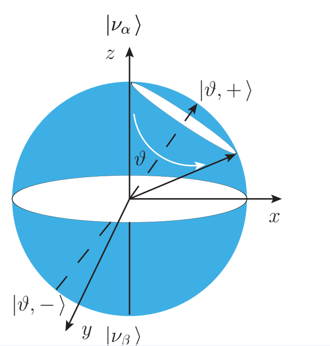
$$C = -\frac{\omega}{2}\cos 2\theta + \frac{1}{2}\sqrt{2}G_F n_e$$

$$A = \frac{\omega}{2}\sin 2\theta, B = 0$$

- For neutrinos $\rightsquigarrow$ absorption negligible.

Visualization tool - The Poincaré sphere

An arbitrary state, $\quad | \ \psi \ \rangle = e^{i\eta} \begin{pmatrix} \cos(\vartheta/2) \ e^{-i\phi/2} \\ \sin(\vartheta/2) \ e^{i\phi/2} \end{pmatrix}$



$\mathbb{H}_\nu(\theta)$ is real ($x - z$ plane)
Half angles used : $\vartheta = 2\theta$
Orthogonal states - antipodal points
$| \ \nu_\alpha \ \rangle$ and $| \ \nu_\beta \ \rangle \Leftrightarrow$ RCP and LCP states
$| \ \vartheta, + \ \rangle$ and $| \ \vartheta, - \ \rangle \Leftrightarrow$ EP states
Oscillation phenomena can be viewed as precession, unitary rotations
MSW effect $\Rightarrow \theta = \pi/4$
complete swapping of flavors
NP rotated into SP (about equator with LP states at anti-podal points)
Polarised states in optics have isomorphic connection with the neutrino states

The 2 flavor neutrino Hamiltonian

Let us examine the form of $\mathbb{H}_\nu$

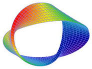
$$\mathbb{H}_\nu = \frac{\omega}{2}[-\cos\vartheta\sigma_z + \sin\vartheta\sigma_x]$$

$$|\,\vartheta,+\,\rangle = \begin{pmatrix}\cos\vartheta/2\\ \sin\vartheta/2\end{pmatrix} \qquad |\,\vartheta,-\,\rangle = \begin{pmatrix}-\sin\vartheta/2\\ \cos\vartheta/2\end{pmatrix}$$

- CP conserved (coeff. of $\sigma_y = 0$), Eigenstates lie on a great circle (intersection of $x - z$ plane with Poincaré sphere)

Eigenstates change sign as ϑ changes from $0 \to 2\pi$

$$|\,\vartheta,\pm\,\rangle = \mp|\,\vartheta+\pi,\mp\,\rangle = -|\,\vartheta+2\pi,\pm\,\rangle$$
$$= \pm|\,\vartheta+3\pi,\mp\,\rangle = |\,\vartheta+4\pi,\pm\,\rangle$$



- Global structure $\rightsquigarrow$ Möbius band
- Expect a phase of ± 1 first noticed in molecular physics in 1958 to appear in the neutrino system. Longuet-Higgins et. al. (1958)

Part I

A short primer on geometric phases

Geometric phases

Why study geometric phases ?

- Unified description of a variety of systems

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- It is an interesting phenomena in quantum mechanics and occurs in many physical systems. It has been tested in many branches of physics - optics, molecular spectroscopy, nuclear magnetic resonance, microwave cavities and so on.

Shapere and Wilczek, *Geometric phases in Physics*, (World Scientific, Singapore, **(1989)**)

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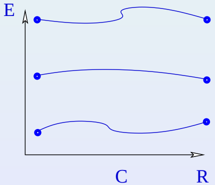
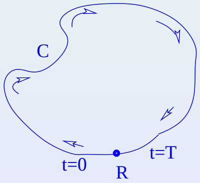
- The greatest value lies in providing a completely new viewpoint to look at the quantum theory.

Anandan, *The geometric phase*, (Nature 360, 307 **(1992)**)

The Berry phase

- Adiabatic closed circuit C in the parameter space
  - Hamiltonian $\mathbb{H}(\mathbf{R}(t))$, $\mathbf{R} \longrightarrow$ external parameters
  - Slow variation of $\mathbf{R}$ (compared to $\hbar/(E_i - E_j)$)
    $|\ \psi(0)\ \rangle = |\ n, \mathbf{R}(0)\ \rangle \Rightarrow |\ \psi(t)\ \rangle \propto |\ n, \mathbf{R}(t)\ \rangle$
    $\rightsquigarrow$ the state clings to an eigenstate (no level crossing) !
    and the basis states (upto a phase) obey
    $\mathbb{H}(\mathbf{R}(t))|\ n, \mathbf{R}(t)\ \rangle = E_n(\mathbf{R}(t))|\ n, \mathbf{R}(t)\ \rangle$
  - Cyclic evolution : at $t = T$, $\mathbf{R}(T) = \mathbf{R}(0)$





$\boxed{|\ \psi(T)\ \rangle = e^{i\varphi}|\ \psi(0)\ \rangle}$ $\rightsquigarrow$ "What is φ after cyclic evolution"

M. Berry, Proc. Roy. Soc. Lond. A392, 45 **(1984)**

The Berry phase

- Adiabatic closed circuit C in the parameter space
- Naive guess : $\varphi = -\int_0^T E_n(\mathbf{R(t)})\mathbf{dt}$ (dynamical phase) is wrong !
- So, what happens to the state $|\ \psi(T)\ \rangle$ under Schrödinger evolution : $i\frac{d}{dt}|\ \psi(t)\ \rangle = \mathbb{H}|\ \psi(t)\ \rangle$

The correct answer

$|\ \psi(T)\ \rangle = e^{i(\delta_n+\gamma_n(C))}|\ \psi(0)\ \rangle$

$\boxed{\delta_n = -\int_0^T E_n(\mathbf{R}(t))dt \text{ (dynamical phase)}}$

$\boxed{\gamma_n = i\oint_C \langle\ n, \mathbf{R}(t)\ |\ \nabla_{\mathbf{R}}\ | n, \mathbf{R}(t)\ \rangle \cdot d\mathbf{R} \text{ (pure geometric phase)}}$

$\boxed{\mathcal{A}_n(\mathbf{R}) = i\langle\ n, \mathbf{R}(t)\ |\ \nabla_{\mathbf{R}}\ | n, \mathbf{R}(t)\ \rangle}$ the Berry connection (like vector potential in parameter space)

$\gamma_n(C) = i\oint_C \mathcal{A}_n(\mathbf{R}) \cdot d\mathbf{R}$ (like AB phase in parameter space)

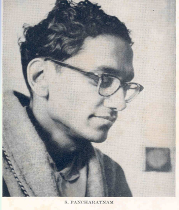
Berry's phase and quantum parallel transport

- J. Samuel and R. Bhandari, Phys. Rev. Lett. 60, 2339 **(1988)**

- Upon removing the dynamical phase,
  $| \phi(t) \rangle = e^{+i \int_0^t \langle \psi(t') | \mathbb{H} | \psi(t') \rangle dt'} | \psi(t) \rangle$
  Schrödinger equation implies the parallel-transport rule for neighbouring states,
  $\Im m \langle \phi(t) | \dot{\phi}(t) \rangle = 0$ (natural connection)
- Non-integrable law, as we go round a closed loop C, $| \phi(T) \rangle$ returns with a changed phase, $| \phi(T) \rangle = e^{i\gamma_n(C)} | n, \mathbf{R}(T) \rangle$ which is quantum geometric phase $\gamma_n(C)$
- $\dot{\gamma}_n(t) = i \langle n, \mathbf{R}(t) | \dot{n}, \mathbf{R}(t) \rangle \neq 0$
- Finally, one gets $\boxed{| \psi(T) \rangle = e^{i(\delta_n + \gamma_n(C))} | \psi(0) \rangle}$

Essential requirements :-

multi-dimensional parameter space ($n \geq 2$) to explore curvature, adiabatic and cyclic evolution of non-degenerate eigenstates



The Pancharatnam phase

S. Pancharatnam, Proc. Ind. Acad. Sci. A44, 247 **(1956)**

M. V. Berry, J. Mod. Opt. 34, 1401 **(1987)**

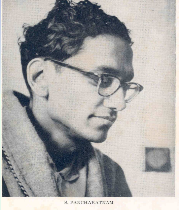
- What would be a natural way to compare the phases of non-orthogonal states ?
- Notion of geometric parallelism from the inner product, $\langle\, \mathcal{A} \,|\, \mathcal{B} \,\rangle$

Reference condition : Pancharatnam's connection or rule

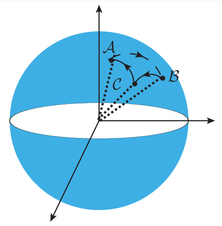
If $\langle\, \mathcal{A} \,|\, \mathcal{B} \,\rangle$ real and positive $\rightsquigarrow$ "in phase" or parallel

$$|||\, \mathcal{A} \,\rangle + |\, \mathcal{B} \,\rangle||^2 = \langle\, \mathcal{A} \,|\, \mathcal{A} \,\rangle + \langle\, \mathcal{B} \,|\, \mathcal{B} \,\rangle + 2|\langle\, \mathcal{A} \,|\, \mathcal{B} \,\rangle| \cos(\mathrm{ph}(\langle\, \mathcal{A} \,|\, \mathcal{B} \,\rangle))$$

- Geometrically, norm of resultant vector is maximum. Physically, interference of superposed beams gives maximum probability (intensity)
- Pancharatnam's connection is both reflexive and symmetric, but not transitive $\rightsquigarrow$ Pancharatnam's phase
- Out of three non-orthogonal rays, if pairwise any two of them are in phase i.e. if $|\, \mathcal{A} \,\rangle$ "in phase" $|\, \mathcal{B} \,\rangle$ and $|\, \mathcal{B} \,\rangle$ "in phase" $|\, \mathcal{C} \,\rangle$ then $|\, \mathcal{C} \,\rangle$ "not in phase" $|\, \mathcal{A} \,\rangle$



The Pancharatnam phase



Pancharatnam's non-integrable phase β

phase of the complex number, $\langle\, \mathcal{A} \,|\, \mathcal{C}\, \rangle \langle\, \mathcal{C} \,|\, \mathcal{B}\, \rangle \langle\, \mathcal{B} \,|\, \mathcal{A}\, \rangle \equiv \mathrm{r} e^{i\beta}$
$=$ half the solid angle Ω subtended by the geodesic triangle $\mathcal{A}\ \mathcal{B}\ \mathcal{C}$ on the Poincaré sphere for a two level system at its center

- Pancharatnam's Phase reflects the curvature of the projective Hilbert space and is independent of any parameterization or slow variation.
- The state does not have to be an eigenstate of $\mathbb{H}$. Pancharatnam's Phase can appear in situations when $\mathbb{H}$ is constant in time.

The Pancharatnam phase and collapses

- Schrödinger Evolution (possibly) interrupted by measurements can lead to Pancharatnam's Phase
- If we take any state and subject it to multiple quantum collapses and bring it back to itself, then the resulting state is given by $|\,\mathcal{A}\,\rangle\langle\,\mathcal{A}\,|\,\mathcal{C}\,\rangle\langle\,\mathcal{C}\,|\,\mathcal{B}\,\rangle\langle\,\mathcal{B}\,|\,\mathcal{A}\,\rangle$ where the phase of the complex number is given by $\Omega/2$.

Essential requirements :-

minimum 3 states (neighbouring ones non-orthogonal) for non-transitivity and exploring the curvature of ray space (which is always curved) and
cyclic projection of states

Geometric phases and neutrinos
A long history

Neutrino flavor oscillations :

Berry's geometric phase :

- N. Nakagawa, Ann. Phys. 179, 145 **(1987)**
- V. A. Naumov, JETP Lett. 54, 185 **(1991)**
- V. A. Naumov, Sov. Phys. JETP Lett. 74, 1 **(1992)**
- V. A. Naumov, Int. Jour. Mod. Phys. D1, 379 **(1992)**
- V. A. Naumov, Phys. Lett. B323, 351 **(1994)**
- X-G. He et. al., Phys. Rev. D72, 053012 **(2005)**

Generalized geometric phases :

- M. Blasone, P. A. Henning and G. Vitiello, Phys. Lett. B466, 262 **(1999)**
- X-B. Wang et. al., Phys. Rev. D63, 053003 **(2001)**
- PM, Phys. Rev. D79, 096013 **(2009)**
- M. Blasone et. al., arXiv:0903.1578 **(2009)**
- PM, arXiv:0907.0562 [hep-ph] **(2009)**

Left-Right spin precession : *considering the spin of neutrinos and systems with intense magnetic fields (sun, supernova)*

- J. Vidal and J. Wudka, Phys. Lett. B259, 473 **(1990)**
- C. Aneziris and J. Schechter, Int. Jour. Mod. Phys. A6, 2375 **(1991)**
- A. Yu. Smirnov, Pisma. Zh. Eksp. Teor. Fiz. 53, 280 **(1991)** [JETP Lett. 53, 291 **(1991)**]
- A. Yu. Smirnov, Phys. Lett. B260, 161 **(1991)**
- E. Kh. Akhmedov, P. I. Krastev and A. Yu. Smirnov, Z. Phys. C - Particles and Fields 52, 701 **(1991)**
- C. Aneziris and J. Schechter, Phys. Rev. D45, 1053 **(1992)**
- J. Schechter, SU-4240-513 **(1992)**
- M. M. Guzzo and J. Bellandi, Phys. Lett. B 294, 243 **(1992)**
- V. M. Aquino, J. Bellandi and M. M. Guzzo, Phys. Scr. 54, 328 **(1996)**

Part II

2 flavor oscillations and the topological phase

Results

Detection : Split beam experiment

Two flavor oscillation formulae

Direct detection of geometric phases

- The key ingredient :

Split-beam experiment

$$||(|\,\psi_1\,\rangle + e^{i\gamma}|\,\psi_2\,\rangle)||^2 = \langle\,\psi_1\,|\,\psi_1\,\rangle + \langle\,\psi_2\,|\,\psi_2\,\rangle + e^{-i\gamma}\langle\,\psi_2\,|\,\psi_1\,\rangle + e^{i\gamma}\langle\,\psi_1\,|\,\psi_2\,\rangle$$

- A beam is split into two parts, which traverse different histories on the Poincaré sphere and finally recombined.
- Main obstacle : One needs a source and detector of neutrinos and the beam has to take two paths between them.
- The refractive index of neutrinos is so small that the focal length of any object in the solar system is astronomical.
- So, we cannot do what is done in optics : use mirrors or lenses to separate and recombine a beam.

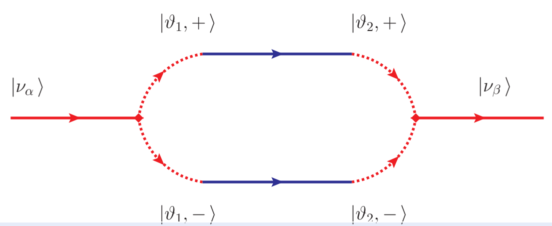
Split-beam experiment in physical space

- Clearly impossible !

Split-beam interference experiment in energy space

Think of oscillations in flavor space as

performing a split beam experiment in energy space by doing quantum collapses along with adiabatic evolution



Split-beam two-path-interferometer

Two flavor neutrino oscillation probability

Start with flavor states $|\ \nu_\alpha\ \rangle$

$|\ \nu_\alpha\ \rangle = \nu_{\alpha+}|\ \vartheta_1, +\ \rangle + \nu_{\alpha-}|\ \vartheta_1, -\ \rangle$

where, $|\ \vartheta_1, \pm\ \rangle$ are the eigenstates of $\mathbb{H}_\nu(\vartheta_1) = [(\sin\vartheta_1)\sigma_x + (-\cos\vartheta_1)\sigma_z]$.

Adiabatic evolution of mass states from $|\ \vartheta_1, \pm\ \rangle$ to $|\ \vartheta_2, \pm\ \rangle$

$$\begin{aligned} |\ \vartheta_1, \pm\ \rangle &\rightarrow e^{-i\mathcal{D}_\pm}|\ \vartheta_2, \pm\ \rangle \quad \text{with} \\ \mathcal{D}_\pm &\simeq \pm\frac{1}{2}\int\sqrt{(\omega\sin\vartheta)^2 + (V_C - \omega\cos\vartheta)^2}dt \end{aligned}$$

Amplitude

$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = \langle\ \nu_\beta\,|\ \mathcal{U}\ |\,\nu_\alpha\ \rangle$ where $\mathcal{U}$ is the unitary evolution operator given by,

$$\mathcal{U} = e^{-i\mathcal{D}_+}|\ \vartheta_2, +\ \rangle\langle\ \vartheta_1, +\ | + e^{-i\mathcal{D}_-}|\ \vartheta_2, -\ \rangle\langle\ \vartheta_1, -\ |\ .$$

Two flavor neutrino oscillation probability

Probability

$\mathcal{P}(\nu_\alpha \to \nu_\beta) = |\mathcal{A}(\nu_\alpha \to \nu_\beta)|^2$

$$\begin{aligned}\mathcal{P}(\nu_\alpha \to \nu_\beta) \quad = \quad & \langle\, \nu_\alpha \,|\, \vartheta_1, + \,\rangle\langle\, \vartheta_2, + \,|\, \nu_\beta \,\rangle\langle\, \nu_\beta \,|\, \vartheta_2, + \,\rangle\langle\, \vartheta_1, + \,|\, \nu_\alpha \,\rangle \\ & + \langle\, \nu_\alpha \,|\, \vartheta_1, - \,\rangle\langle\, \vartheta_2, - \,|\, \nu_\beta \,\rangle\langle\, \nu_\beta \,|\, \vartheta_2, - \,\rangle\langle\, \vartheta_1, - \,|\, \nu_\alpha \,\rangle \\ & + [\langle\, \nu_\alpha \,|\, \vartheta_1, - \,\rangle e^{i\mathcal{D}_-}\langle\, \vartheta_2, - \,|\, \nu_\beta \,\rangle\langle\, \nu_\beta \,|\, \vartheta_2, + \,\rangle e^{-i\mathcal{D}_+}\langle\, \vartheta_1, + \,|\, \nu_\alpha \,\rangle + \mathrm{c.c.}]\end{aligned}$$

- cross-terms (upon removing the dynamical phase) are connected to the two path interferometer in energy space
- can be viewed as closed loop quantum collapses with intermediate adiabatic evolutions
- great circle in $x - z$ plane.

Interference terms as collapses + adiabatic evolution

Cross-terms

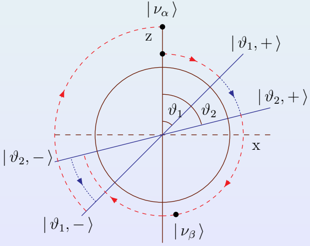
$$\langle\, \nu_\alpha \,|\, \vartheta_1, - \,\rangle\langle\, \vartheta_2, - \,|\, \nu_\beta \,\rangle\langle\, \nu_\beta \,|\, \vartheta_2, + \,\rangle\langle\, \vartheta_1, + \,|\, \nu_\alpha \,\rangle \equiv r e^{i\text{ß}}$$

Appearance terms ($\nu_\alpha \to \nu_\beta$)

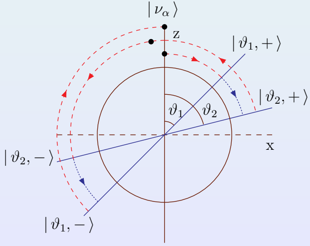
$$\text{ß} = \pi$$

Disappearance terms ($\nu_\alpha \to \nu_\alpha$)

$$\text{ß} = 0$$



(a)



(b)

Compare with the standard expressions

Transition probability

$$\begin{aligned}\mathcal{P}(\nu_e \to \nu_\mu) &= \mathbb{U}^\star_{e+}(\theta_1)\mathbb{U}_{\mu+}(\theta_2)\mathbb{U}^\star_{\mu+}(\theta_2)\mathbb{U}_{e+}(\theta_1)\\ &+ \mathbb{U}^\star_{e-}(\theta_1)\mathbb{U}_{\mu-}(\theta_2)\mathbb{U}^\star_{\mu-}(\theta_2)\mathbb{U}_{e-}(\theta_1)\\ &[\mathbb{U}^\star_{e-}(\theta_1)e^{i\mathcal{D}_-}\mathbb{U}_{\mu-}(\theta_2)\mathbb{U}^\star_{\mu+}(\theta_2)e^{-i\mathcal{D}_+}\mathbb{U}_{e+}(\theta_1) + \text{c.c.}]\end{aligned}$$

For the 2×2 case, $\mathbb{U}(\theta)$ is a real orthogonal rotation matrix given by,

$$\mathbb{U}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$\mathcal{P}(\nu_e \to \nu_\mu) = \cos^2\theta_1 \sin^2\theta_2 + \sin^2\theta_1\cos^2\theta_2$
$+ [2\cos(\mathcal{D}_+ - \mathcal{D}_-)](-\sin\theta_1)\cos\theta_2\sin\theta_2\cos\theta_1$
topological phase $=\pi$
$\mathcal{P}(\nu_e \to \nu_e) = \cos^2\theta_1\cos^2\theta_2 + \sin^2\theta_1\sin^2\theta_2$
$+ [2\cos(\mathcal{D}_+ - \mathcal{D}_-)]\sin\theta_1\cos\theta_2\sin\theta_2\cos\theta_1$
topological phase $=0$, in accord with Unitarity, $\mathcal{P}(\nu_e\to\nu_\mu)+\mathcal{P}(\nu_e\to\nu_e)=1$

Standard expressions : vacuum and constant density matter

In vacuum for $\theta_1 = \theta_2 = \theta$

$$\begin{aligned} \mathcal{P}(\nu_e \to \nu_\mu) &= \sin^2 2\theta \sin^2 \frac{\delta m^2 l}{4E} \quad \text{and} \\ \mathcal{P}(\nu_e \to \nu_e) &= 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 l}{4E} \ , \end{aligned}$$

where in the ultra-relativistic limit, we can use $t \simeq l$ and $p \simeq E$ leading to $\mathcal{D}_\pm = \pm \delta m^2 l / 2E$ for the vacuum case ($V_C = V_N = 0$).

In matter of constant density

replace θ and δm^2 by θ^m and $(\delta m^2)^m$

Hence our result is consistent with the standard neutrino oscillation formulation and it provides a clear geometric interpretation of the phenomenon of neutrino oscillations.

Part III

Imprint of the CPV phase

Imprint of the CPV phase

Sources of the CPV phase

Neglecting absorption

$$\mathbb{H} = \begin{bmatrix} z & x-iy \\ x+iy & -z \end{bmatrix} + r_0\mathbb{I}_2 = e^{-i\int r_0 dt}\begin{bmatrix} -\cos\vartheta & \sin\vartheta\ e^{-i\varphi} \\ \sin\vartheta\ e^{i\varphi} & \cos\vartheta \end{bmatrix}$$

Medium	x	y	z
Vacuum	$(\frac{\omega}{2})\sin\vartheta$	0	-$(\frac{\omega}{2})\cos\vartheta$
Ordinary medium+SI	$(\frac{\omega}{2})\sin\vartheta$	0	-$(\frac{\omega}{2})\cos\vartheta + \frac{V_C}{2}$
Ordinary medium+NSI	$\mathcal{R}e\left((\frac{\omega}{2})\sin\vartheta + \frac{\epsilon_{ey}}{2}\right)$	$\mathcal{I}m\left((\frac{\omega}{2})\sin\vartheta + \frac{\epsilon_{ey}}{2}\right)$	-$(\frac{\omega}{2})\cos\vartheta + \frac{V_C}{2} + \frac{(\epsilon_{ee}-\epsilon_{yy})}{2}$
Neutrino backgrounds+SI	$\mathcal{R}e\left((\frac{\omega}{2})\sin\vartheta + \frac{B_{ey}}{2}\right)$	$\mathcal{I}m\left((\frac{\omega}{2})\sin\vartheta + \frac{B_{ey}}{2}\right)$	-$(\frac{\omega}{2})\cos\vartheta + \frac{V_C}{2} + \frac{B}{2}$

Table: The three independent elements of $\mathbb{H}$ in different kinds of media.

Pictorial depiction of the cross terms

$$\langle\, \psi \,|\, \vartheta_1, - \,\rangle\langle\, \vartheta_2, - \,|\, \chi \,\rangle\langle\, \chi \,|\, \vartheta_2, + \,\rangle\langle\, \vartheta_1, + \,|\, \psi \,\rangle \equiv r e^{i\text{ß}}$$

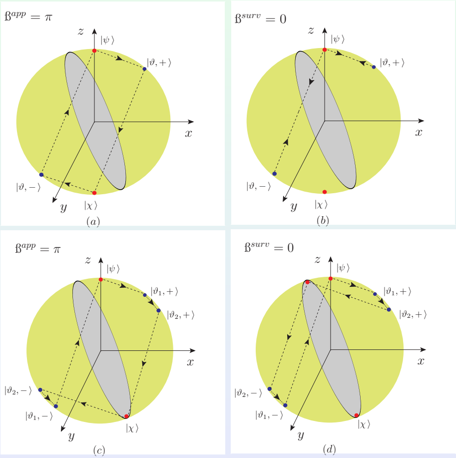


Figure: CPC situation

Pictorial depiction of the cross terms

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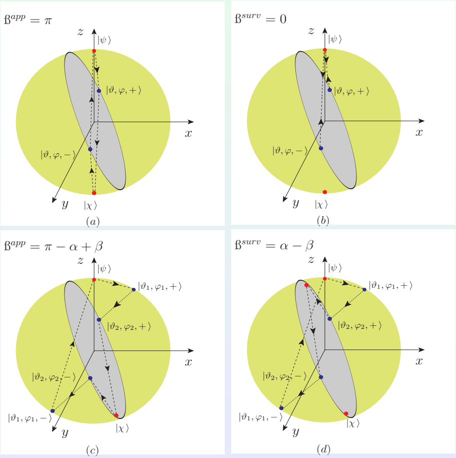

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- Using Pancharatnam's ideas in the two flavor neutrino oscillations, we show that there exists a topological phase at the probability level. The standard formalism is in fact a realization of the Pancharatnam phase.
- The non-trivial phase of π and anholonomy is linked to encircling of a singular point in the ray space.
- We also made a connection to the π phase obtained first in the context of molecular physics in 1958.
- This phase remains irrespective of adiabatic evolution or propagation of neutrinos in vacuum and is a robust quantity.
- The topological robustness can be destroyed once we invoke CP violation under suitable conditions.

Part IV

Extras

Characteristic scales and sensitivity to δm^2

Experiment	L(km)	E(GeV)	δm^2(eV^2)
Solar	10^7	10^{-3}	10^{-10}
Atmospheric	$10^1 - 10^4$	$10^{-1} - 10^2$	$10^{-1} - 10^{-4}$
Supernova	10^7	10^{-2}	10^{-9}
Reactor	$10^{-1} - 10^1$	10^{-3}	$10^{-2} - 10^{-3}$
Accelerator	10^{-1}	$10^{-1} - 10^1$	≥ 0.1
LBL Accelerator	$10^2 - 10^3$	10^1	$10^{-2} - 10^{-3}$

Table: Characteristic values of L and E for various neutrino experiments and sources. Note that if E is in units of MeV and L in units of m, we will obtain the same value for δm^2 that can be probed. Thus the pair (L, E) can be in the units (km, GeV) or $(\mathrm{m},$ MeV) and both sets will give the same sensitivity to the value of δm^2 in eV^2.

What about incoherent scattering effects ?

In most practical situations, the incoherent scattering cross-section of neutrinos with matter is very small $\Rightarrow$ Sustained coherence seen even over astrophysical length scales !

Medium	ρ (g/cm^3)	$l_{mfp} = 10^{38}/(N_{Avo}\rho Y_f M E)$ (cm)
Earth core	~ 10	$\sim 10^{13}$-10^{19}
Solar core	~ 100	$\sim 10^{12}$-10^{18}
Supernova core	$\sim 10^{14}$	~ 1-10^{6}

Table: Examples of different density regions that are accessible to observations and the value of mean free path taking the target mass to be $M = 1$ GeV (1 MeV) and neutrino energy to be $E = 1$ GeV (1 MeV).

Neutrino refraction - Imaginary part

- For $\nu_e - e$ CC scattering ($E's << M_W$), upto G_F^2, using optical thm.,

$$\begin{aligned}\Re e f(0) + i\Im m f(0) &= -\frac{E}{2\pi n_e}(V_C) + i\Im m f(0) \\ &= -\frac{E}{2\pi n_e}(V_C) + i\frac{E}{4\pi}\sigma_T \\ &= -\frac{E}{2\pi n_e}(V_C) + i\frac{E}{4\pi}\frac{1}{n_e l_{mfp}}\end{aligned}$$

- At zero temperature, total refractive index is

$$n_{refr} - 1 \simeq -\frac{(\sqrt{2}G_F n_e)}{E} + i\frac{1}{2E}n_e\sigma_T$$

- Real part $\Re e[n_{refr}] \propto G_F$ describes coherent interference of propagating neutrinos
- Imaginary part $\Im m[n_{refr}] \propto G_F^2 q^2$ is responsible for incoherent depletion of neutrinos from original coherent state
- In most situations, absorption is negligible, $\Re e[n_{refr}] >> \Im m[n_{refr}]$ since neutrinos interact via weak interactions.

The π anholonomy

Non-integrable phases of ± 1 can arise in BO approximation in molecular physics (e^- spin neglected and $\mathbb{H}$ real)

Longuet-Higgins et al., Proc. Roy. Soc. Lond. A244,1 **(1958)**, Herzberg and Longuet-Higgins, Disc. Faraday Soc. 35, 77 **(1963)**

Consider a real 2×2 Hamiltonian

$$\mathbb{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix}$$

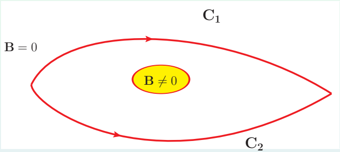
Degenerate eigenvalues $\Rightarrow$ 2 conditions

$h_{11} - h_{22} = 0$ and $h_{12} = 0$ must be satisfied
if $x = (h_{11} - h_{22})/2$ and $y = h_{12}$ then $E_{\pm} = E_0 \pm \sqrt{x^2 + y^2}$ (double cone) where $E_0 = (h_{11} + h_{22})/2$ (crossing energy)

- Encircling the degeneracy $\Rightarrow$ eigenstate picks up a minus sign as we vary θ from $0 \rightarrow 2\pi$ continuously.
- n-times $\Rightarrow$ phase is $(-1)^n$
- Degeneracy can be accidental or generic (need not be connected to symmetry)

The Aharonov-Bohm phase

Aharonov and Bohm, Phys. Rev. 115, 485 **(1959)**



- Importance of vector potential
- Even if the magnetic field $\mathbf{B} = 0$ in a certain region, the vector potential $\mathbf{A}$ is non-zero and that causes a non-trivial phase.
- Encircling the infinite flux tube leads to anholonomy and the phase is always quantized to π.

An example : no varied parameters in $\mathbb{H}$

Consider a constant Hamiltonian and a general state

$\mathbb{H} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $|\, \psi(0)\, \rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$ upon evolving $|\, \psi(t)\, \rangle = \begin{pmatrix} \cos\theta/2 e^{-it/2} \\ \sin\theta/2 e^{it/2} \end{pmatrix}$

Dynamical phase

$\delta = \int_0^T \langle\, \psi \,|\, \mathbb{H} \,|\, \psi\, \rangle dt = T/2(\cos^2\theta/2 - \sin^2\theta/2) = T/2\cos\theta$

Exact solution gives net phase of π for $t = 2\pi$

because $|\, \psi(2\pi)\, \rangle = -|\, \psi(0)\, \rangle = e^{i2\pi/2}|\, \psi(0)\, \rangle$

Missing piece is the geometric phase

$\pi(1-\cos\theta) = \Omega/2$ where Ω is the solid angle subtended by a loop of fixed θ when $t = 2\pi$ i.e. $\Omega = 2\pi(1-\cos\theta)$

Thus, a geometric phase appears irrespective of presence of any variable parameters in Hamiltonian.

Sun as a lens ?

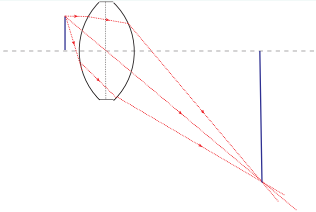
- Can we make devices similar to the optical devices using reflective and refractive property of neutrinos ?

Sun as a lens ?

- Can we make devices similar to the optical devices using reflective and refractive property of neutrinos ?
- If we take Sun as a lens, then the focal length is given by

$$f = \frac{1}{2}\frac{R_\odot}{(n_{refr} - 1)}$$

Lens Maker's formula (tiny n_{refr} limit)



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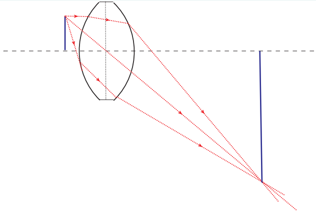

- *For 10 MeV neutrinos passing through Sun with density $\rho = 150$ g cm^{-3}, one gets the focal length to be around $10^{18} R_\odot \sim 10^5$ size of our Galaxy.*

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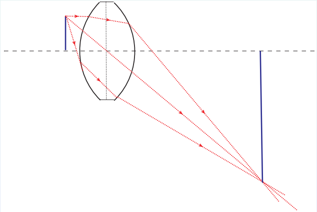

- *For* 10 *MeV neutrinos passing through Sun with density* $\rho = 150$ *g cm*$^{-3}$*, one gets the focal length to be around* $10^{18} R_\odot \sim 10^5$ *size of our Galaxy.*
- Potentially observable effect of small refractive index is via neutrino oscillations.

Extrinsic CPV phases

Nonstandard Interactions and their impact on coherent forward scattering

$$\mathcal{L} = \sum_{f;\alpha,\beta} 4\frac{G_F}{\sqrt{2}}\bar{\nu}_{\alpha L}\gamma^{\mu}\nu_{\beta L}(\epsilon^{fL}_{\alpha\beta}\bar{f}_L\gamma_\mu f_L + \epsilon^{fR}_{\alpha\beta}\bar{f}_R\gamma_\mu f_R)$$

$$\epsilon_{\alpha\beta} = \sum_{f=e,u,d}\frac{n_f}{n_e}\epsilon^f_{\alpha\beta}$$

where $f = e, p, n$ and $\alpha, \beta = e, \mu, \tau$. $\Lambda_L >> \Lambda_{np} > \Lambda_{ew} = G_F^{-1/2}$. $\epsilon^f = \epsilon^{fL} + \epsilon^{fR}$.

Neutrino backgrounds in dense Supernovae, $|\ \nu_b\ \rangle = \gamma_e|\ \nu_e\ \rangle + \gamma_y|\ \nu_y\ \rangle$

$$\varrho_{\mathbf{p}} = |\ \nu_b\ \rangle\langle\ \nu_b\ | = \begin{bmatrix} |\gamma_e|^2 & \gamma_e\gamma_y^\star \\ \gamma_e^\star\gamma_y & |\gamma_y|^2 \end{bmatrix}$$

$$\begin{aligned} B &= \sqrt{2}G_F\int d^3\mathbf{q}(1-\cos\theta_{\mathbf{p}\,\mathbf{q}}) \\ &\quad [(\varrho_{\mathbf{q}}-\bar{\varrho}_{\mathbf{q}})_{ee} - (\varrho_{\mathbf{q}}-\bar{\varrho}_{\mathbf{q}})_{yy}] \\ B_{ey} &= \sqrt{2}G_F\int d^3\mathbf{q}(1-\cos\theta_{\mathbf{p}\,\mathbf{q}})[(\varrho_{\mathbf{q}}-\bar{\varrho}_{\mathbf{q}})_{ey}] \\ B_{ye} &= \sqrt{2}G_F\int d^3\mathbf{q}(1-\cos\theta_{\mathbf{p}\,\mathbf{q}})[(\varrho_{\mathbf{q}}-\bar{\varrho}_{\mathbf{q}})_{ye}] \end{aligned}$$