

Topological phase in two flavor neutrino oscillations :

A new interpretation from geometry

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IACS, Kolkata, 4 December 2009

Acknowledgements

Thanks to Joseph Samuel and Supurna Sinha.

Outline of the introduction

Introduction

- Neutrino oscillations

- Experiments

- Data from oscillation and other experiments

- Oscillations as birefringence

Two flavor case

- Why two flavors ?

- Dispersion relation in vacuum and media

- Neutrino refraction in vacuum and media

Neutrinos and analogous two state systems

- Effects in optics and their counterparts in the neutrino system

- Visualization tool - The Poincaré sphere

Introduction to neutrino oscillations

- Anomalies in the neutrino sector

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Neutrino **flavor** oscillations among the 3 flavors of light active neutrinos (conserving L)

- In SM, neutrinos are massless \rightsquigarrow can not oscillate

Neutrino flavor oscillations is the **only firm evidence in favor of physics beyond the Standard Model** of particle physics, even though there are other strong indications (BAU, DM, DE, ..)

Information gleaned from experiments

- Experiments observing neutrinos from cosmic ray interactions in the atmosphere, sun, reactor, terrestrial experiments reveal that neutrinos undergo a change of flavor
 - Atmospheric expts. $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ (SuperK)
 - Solar expts. $\nu_e \rightarrow \nu_\mu$ or ν_τ (Homestake, SAGE, GALLIUM, Kamiokande, SuperK, SNO, Borexino)
 - Reactor anti-nu expts. $\bar{\nu}_e \rightarrow \bar{\nu}_x$ (KamLAND, CHOOZ, LSND, MiniBooNE)
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Future expts. in, R. Gandhi, PM and S. Uma Sankar, INO/HRI/2005/03 (2005)

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Different scales of energy E and path lengths L allow for different sensitivities to mass-squared differences in vacuum or in matter 

- Very wide range for tunable parameters L, E .

Present data

- 3 flavors $\nu_e, \nu_\mu, \nu_\tau \Rightarrow$ 9 leptonic mass and mixing parameters

PMNS mixing matrix contains $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_2, \alpha_3$

$$\mathbb{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times$$

$$\text{diag}[1, e^{i\alpha_2/2}, e^{i\alpha_3/2}]$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$.

M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep. 460, 1 (2008),

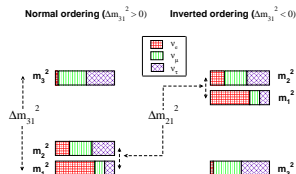
R. Z. Funchal, talk at ν 2008

Present data

- Mass and mixing parameters
- Low energy oscillation data -
 - 3 mixing angles :

$\theta_{12} \approx 32^\circ, \theta_{23} \approx 45^\circ, \theta_{13} \lesssim 10^\circ \text{ (upper bound)}$
 - 2 mass-squared differences :

$\Delta m_{21}^2 \simeq 7.7 \times 10^{-5} eV^2 \text{ and } |\Delta m_{31}^2| \simeq 2.5 \times 10^{-3} eV^2$
 - $\Delta m_{\odot}^2 / \Delta m_{atm}^2 \sim 0.03$
 - Solar data : $\Delta m_{\odot}^2 > 0$
 - Sign of Δm_{31}^2 ?
 - Dirac CP phase δ ?



Present data

- Mass and mixing parameters
- Absolute value of neutrino masses - inaccessible via oscillation
 - β -decay experiments,

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2},$$

$m_\beta < 1.8 \text{ eV}$

 (Mainz+Troitsk)
 - $0\nu\beta\beta$ experiments (sensitive to Majorana nature),

$$m_{\beta\beta} = |\sum_i U_{ei}^2 m_i| = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{i\alpha_3} m_3|,$$

$m_{\beta\beta} = 0.16 - 0.52(0 - 0.25) \text{ eV}$

 (Heidelberg-Moscow
(Cuoricino))
 - Cosmology, $\Omega_\nu \propto \sum = \sum_i m_i$,

$\sum < 1.3 \text{ eV}$

 (WMAP5)
- Two Majorana phases (α_2, α_3) - inaccessible via oscillation
 $0\nu\beta\beta$ experiments

Why do the neutrinos oscillate ?

- Neutrinos are produced and detected via weak interaction

Weak (flavor) eigenstates differ from the stationary (mass) states of the Hamiltonian, infact they are linear superpositions of the stationary mass states

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which get modified due to CC potential for $\nu_e - e$ coherent forward scattering.

- Incoherent scattering cross-section is usually negligible \Rightarrow sustained coherence over astrophysical length scales 

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- Conventionally, atmospheric data can be explained by Δm_{32}^2 and θ_{13} and solar data by Δm_{21}^2 and θ_{12} .

Dispersion relation for two flavor neutrinos

- In the ultra-relativistic limit, the relativistic dispersion relation

In vacuum, neutrinos obey

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$$

Assume equal and fixed momenta, $p_1 = p_2 = p$ (monochromatic)

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- Two flavor neutrinos \rightsquigarrow Two state quantum system
- Hilbert space of this system can be mapped onto a Bloch sphere (analogous to Poincaré sphere in optics)
- In vacuum, the mass-squared difference and mixing between the two neutrinos leads to flavor oscillations i.e.

$$\mathbb{H}_{fl} = \mathbb{U}^\dagger(\theta) \mathbb{H}_m \mathbb{U}(\theta) \text{ is not diagonal.}$$

Inclusion of matter effects (SM interactions)

In ordinary matter

$$\mathbb{H}_\nu = \left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{V_C}{2} + V_N \right) \mathbb{I} + \frac{1}{2} \begin{pmatrix} V_C - \omega \cos 2\theta & \omega \sin 2\theta \\ \omega \sin 2\theta & -(V_C - \omega \cos 2\theta) \end{pmatrix}$$

- $V_C = \sqrt{2}G_F n_e$ and $V_N = -\sqrt{2}G_F n_n/2$ are the SM induced potentials due to neutrino matter (e, n, p) interactions and $\omega = \delta m^2/2p$
vacuum case : $V_C, V_N = 0$

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Wolfenstein (1978), Mikheev and Smirnov (1985)

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vacuum case : $V_C, V_N = 0$
- Most dramatic effect is the MSW resonance due to vanishing diagonal terms
- Absence of FCNC \rightsquigarrow if vacuum mixing and mass-splitting is zero, then **matter does not really matter** (in oscillations).

Wolfenstein (1978), Mikheev and Smirnov (1985)

Neutrino refraction in vacuum and media

L. Wolfenstein, Phys. Rev. D 17, 2367 (1978), P. Langacker and Liu, Phys. Rev. D 46, 4140 (1992)

- In vacuum, neutrino refraction arises due to the $m^2/2E^2$ term

$$n_{refr} - 1 = \frac{p}{E} - 1 \simeq -\frac{m^2}{2E^2}$$

- In medium, one can use coherent forward scattering amplitudes $f(0)$ to compute the n_{refr} ,

$$n_{refr} - 1 \simeq -\frac{m^2}{2E^2} + \frac{2\pi}{E^2} n_e f(0)$$

- For $\nu_e - e$ CC scattering ($E' s \ll M_W$), to leading order in G_F ,

$$f(0) = -\frac{E}{2\pi n_e} (V_C)$$

- At zero temperature, $n_{refr} \propto n_e$, neglecting vacuum term,

$$\begin{aligned} n_{refr} - 1 &\simeq -\frac{(\sqrt{2}G_F n_e)}{E} \\ &= -7.6 \times 10^{-14} Y_e \frac{\rho}{[\text{g cm}^{-3}]} \frac{[\text{eV}]}{E} \end{aligned}$$

Effects in optics and their counterparts in the neutrino system

Effect of medium can be described in terms of

$$\mathbb{H} = D\mathbb{I} + A\sigma_x + B\sigma_y + C\sigma_z$$

D just gives an overall phase, while A, B, C generate non-trivial optical effects

Optical effects

- Circular Birefringence (Optical activity)
 $C, D \neq 0$ while $A, B = 0$

- Linear Birefringence (Wave plate)
 $A, D \neq 0$ while $B, C = 0$

- Elliptic Birefringence (Quartz plate)
 $A, B, C, D \neq 0$ (most general)

- Dichroism (absorptive effect)

\mathbb{H} need not be Hermitian

Neutrino oscillations

- Oscillations in vacuum
 \equiv Elliptic birefringence

$$A = \frac{\omega}{2} \sin 2\theta, B = 0, C = -\frac{\omega}{2} \cos 2\theta$$

- Oscillations in normal matter + SM
 \equiv Elliptic birefringence

$$C = -\frac{\omega}{2} \cos 2\theta + \frac{1}{2} \sqrt{2} G_F n_e$$

$$A = \frac{\omega}{2} \sin 2\theta, B = 0$$

- For neutrinos \rightsquigarrow absorption negligible.

Visualization tool - The Poincaré sphere

An arbitrary state, $|\psi\rangle = e^{i\eta} \begin{pmatrix} \cos(\vartheta/2) e^{-i\phi/2} \\ \sin(\vartheta/2) e^{i\phi/2} \end{pmatrix}$

 $\mathbb{H}_\nu(\theta)$ is real ($x - z$ plane)

Half angles used : $\vartheta = 2\theta$

Orthogonal states - antipodal points

$$|\nu_\alpha\rangle \text{ and } |\nu_\beta\rangle \Leftrightarrow \text{RCP and LCP states}$$
 $|\vartheta, +\rangle$ and $|\vartheta, -\rangle \Leftrightarrow$ EP states

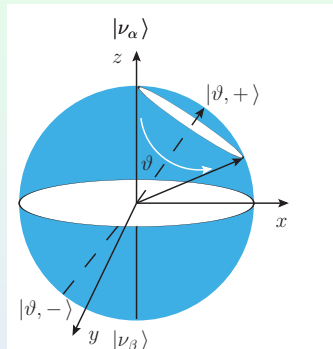
Oscillation phenomena can be viewed as precession, unitary rotations

MSW effect $\Rightarrow \theta = \pi/4$

complete swapping of flavors

NP rotated into SP (about equator with LP states at anti-podal points)

Polarised states in optics have isomorphic connection with the neutrino states



The 2 flavor neutrino Hamiltonian

Let us examine the form of \mathbb{H}_ν

$$\mathbb{H}_\nu = \frac{\omega}{2} [-\cos \vartheta \sigma_z + \sin \vartheta \sigma_x]$$

$$|\vartheta, +\rangle = \begin{pmatrix} \cos \vartheta/2 \\ \sin \vartheta/2 \end{pmatrix} \quad |\vartheta, -\rangle = \begin{pmatrix} -\sin \vartheta/2 \\ \cos \vartheta/2 \end{pmatrix}$$

- CP conserved (coeff. of $\sigma_y = 0$), Eigenstates lie on a great circle (intersection of $x - z$ plane with Poincaré sphere)

Eigenstates change sign as ϑ changes from $0 \rightarrow 2\pi$

$$\begin{aligned} |\vartheta, \pm\rangle &= \mp |\vartheta + \pi, \mp\rangle = -|\vartheta + 2\pi, \pm\rangle \\ &= \pm |\vartheta + 3\pi, \mp\rangle = |\vartheta + 4\pi, \pm\rangle \end{aligned}$$



- Global structure \rightsquigarrow Möbius band
- Expect a phase of ± 1 first noticed in molecular physics in 1958 to appear in the neutrino system.

Part I

A short primer on geometric phases

Geometric phases

Why study geometric phases ?

Geometric phases - the two avatars

Geometric phases and neutrinos

Why study geometric phases ?

- Unified description of a variety of systems

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- Unified description of a variety of systems
- It is an interesting phenomena in quantum mechanics and occurs in many physical systems. It has been tested in many branches of physics - optics, molecular spectroscopy, nuclear magnetic resonance, microwave cavities and so on.

Shapere and Wilczek, *Geometric phases in Physics*, (World Scientific, Singapore, (1989))

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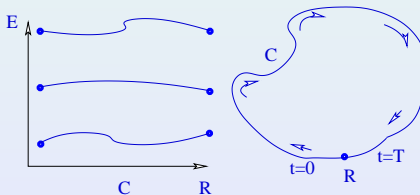
Shapere and Wilczek, *Geometric phases in Physics*, (World Scientific, Singapore, (1989))

- The greatest value lies in providing a completely new viewpoint to look at the quantum theory.

Anandan, *The geometric phase*, (Nature 360, 307 (1992))

The Berry phase

- Adiabatic closed circuit C in the parameter space
- Hamiltonian $\mathbb{H}(\mathbf{R}(t))$, $\mathbf{R} \rightarrow$ external parameters
- Slow variation of \mathbf{R} (compared to $\hbar/(E_i - E_j)$)
 $|\psi(0)\rangle = |n, \mathbf{R}(0)\rangle \Rightarrow |\psi(t)\rangle \propto |n, \mathbf{R}(t)\rangle$
 \rightsquigarrow the state clings to an eigenstate (no level crossing) !
 and the basis states (upto a phase) obey
 $\mathbb{H}(\mathbf{R}(t))|n, \mathbf{R}(t)\rangle = E_n(\mathbf{R}(t))|n, \mathbf{R}(t)\rangle$
- Cyclic evolution : at $t = T$, $\mathbf{R}(T) = \mathbf{R}(0)$



$$|\psi(T)\rangle = e^{i\varphi} |\psi(0)\rangle \rightsquigarrow \text{"What is } \varphi \text{ after cyclic evolution"}$$



The Berry phase

- Adiabatic closed circuit C in the parameter space
- Naive guess : $\varphi = - \int_0^T E_n(\mathbf{R}(t)) dt$ (dynamical phase) is wrong !
- So, what happens to the state $|\psi(T)\rangle$ under Schrödinger evolution :

$$i \frac{d}{dt} |\psi(t)\rangle = \mathbb{H} |\psi(t)\rangle$$



The correct answer

$$|\psi(T)\rangle = e^{i(\delta_n + \gamma_n(C))} |\psi(0)\rangle$$

$$\delta_n = - \int_0^T E_n(\mathbf{R}(t)) dt \text{ (dynamical phase)}$$

$$\gamma_n = i \oint_C \langle n, \mathbf{R}(t) | \nabla_{\mathbf{R}} | n, \mathbf{R}(t) \rangle \cdot d\mathbf{R} \text{ (pure geometric phase)}$$

$$\mathcal{A}_n(\mathbf{R}) = i \langle n, \mathbf{R}(t) | \nabla_{\mathbf{R}} | n, \mathbf{R}(t) \rangle \text{ the Berry connection (like vector potential in parameter space)}$$

$$\gamma_n(C) = i \oint_C \mathcal{A}_n(\mathbf{R}) \cdot d\mathbf{R} \text{ (like AB phase in parameter space)}$$

Berry's phase and quantum parallel transport

- J. Samuel and R. Bhandari, Phys. Rev. Lett. 60, 2339 (1988)

- Upon removing the dynamical phase,

$$| \phi(t) \rangle = e^{+i \int_0^t \langle \psi(t') | \mathbb{H} | \psi(t') \rangle dt'} | \psi(t) \rangle$$
 Schrödinger equation implies the parallel-transport rule for neighbouring states,

$$\Im m \langle \phi(t) | \dot{\phi}(t) \rangle = 0 \text{ (natural connection)}$$
- Non-integrable law, as we go round a closed loop C , $| \phi(T) \rangle$ returns with a changed phase, $| \phi(T) \rangle = e^{i\gamma_n(C)} | n, \mathbf{R}(T) \rangle$ which is quantum geometric phase $\gamma_n(C)$
- $\dot{\gamma}_n(t) = i \langle n, \mathbf{R}(t) | \dot{n}, \mathbf{R}(t) \rangle \neq 0$
- Finally, one gets $| \psi(T) \rangle = e^{i(\delta_n + \gamma_n(C))} | \psi(0) \rangle$

Essential requirements :-

multi-dimensional parameter space ($n \geq 2$) to explore curvature, adiabatic and cyclic evolution of non-degenerate eigenstates



The Pancharatnam phase

S. Pancharatnam, Proc. Ind. Acad. Sci. A44, 247 (1956)

M. V. Berry, J. Mod. Opt. 34, 1401 (1987)

- What would be a natural way to compare the phases of non-orthogonal states ?
- Notion of geometric parallelism from the inner product, $\langle \mathcal{A} | \mathcal{B} \rangle$

Reference condition : Pancharatnam's connection or rule

If $\langle \mathcal{A} | \mathcal{B} \rangle$ real and positive \rightsquigarrow "in phase" or parallel

$$|| | \mathcal{A} \rangle + | \mathcal{B} \rangle ||^2 = \langle \mathcal{A} | \mathcal{A} \rangle + \langle \mathcal{B} | \mathcal{B} \rangle + 2|\langle \mathcal{A} | \mathcal{B} \rangle| \cos(\text{ph}(\langle \mathcal{A} | \mathcal{B} \rangle))$$

- Geometrically, norm of resultant vector is maximum. Physically, interference of superposed beams gives maximum probability (intensity)
- Pancharatnam's connection is both reflexive and symmetric, but not transitive \rightsquigarrow Pancharatnam's phase
- Out of three non-orthogonal rays, if pairwise any two of them are in phase i.e. if $| \mathcal{A} \rangle$ "in phase" $| \mathcal{B} \rangle$ and $| \mathcal{B} \rangle$ "in phase" $| \mathcal{C} \rangle$ then $| \mathcal{C} \rangle$ "not in phase" $| \mathcal{A} \rangle$



phases ?

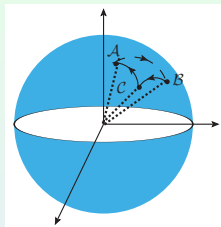
Geometric phases - the two avatars

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The Pancharatnam phase



Pancharatnam's non-integrable phase β

phase of the complex number, $\langle \mathcal{A} | \mathcal{C} \rangle \langle \mathcal{C} | \mathcal{B} \rangle \langle \mathcal{B} | \mathcal{A} \rangle \equiv \text{re}^{i\beta}$
 = half the solid angle Ω subtended by the geodesic triangle $\mathcal{A} \mathcal{B} \mathcal{C}$ on the Poincaré sphere for a two level system at its center

- Pancharatnam's Phase reflects the curvature of the projective Hilbert space and is independent of any parameterization or slow variation.
- The state does not have to be an eigenstate of \mathbb{H} . Pancharatnam's Phase can appear in situations when \mathbb{H} is constant in time. 

The Pancharatnam phase and collapses

- Schrödinger Evolution (possibly) interrupted by measurements can lead to Pancharatnam's Phase
- If we take any state and subject it to multiple quantum collapses and bring it back to itself, then the resulting state is given by $|\mathcal{A}\rangle\langle\mathcal{A}|\mathcal{C}\rangle\langle\mathcal{C}|\mathcal{B}\rangle\langle\mathcal{B}|\mathcal{A}\rangle$ where the phase of the complex number is given by $\Omega/2$.

Essential requirements :-

minimum 3 states (neighbouring ones non-orthogonal) for non-transitivity and exploring the curvature of ray space (which is always curved) and
cyclic projection of states

Geometric phases and neutrinos

A long history

Neutrino flavor oscillations :

Berry's geometric phase :

- N. Nakagawa, Ann. Phys. 179, 145 (1987)
- V. A. Naumov, JETP Lett. 54, 185 (1991)
- V. A. Naumov, Sov. Phys. JETP Lett. 74, 1 (1992)
- V. A. Naumov, Int. Jour. Mod. Phys. D1, 379 (1992)
- V. A. Naumov, Phys. Lett. B323, 351 (1994)
- X-G. He et. al., Phys. Rev. D72, 053012 (2005)

Generalized geometric phases :

- M. Blasone, P. A. Henning and G. Vitiello, Phys. Lett. B466, 262 (1999)
- X-B. Wang et. al., Phys. Rev. D63, 053003 (2001)
- PM, Phys. Rev. D79, 096013 (2009)
- M. Blasone et. al., arXiv:0903.1578 (2009)
- PM, arXiv:0907.0562 [hep-ph] (2009)

Left-Right spin precession : *considering the spin of neutrinos and systems with intense magnetic fields (sun, supernova)*

- J. Vidal and J. Wudka, Phys. Lett. B259, 473 (1990)
- C. Aneziris and J. Schechter, Int. Jour. Mod. Phys. A6, 2375 (1991)
- A. Yu. Smirnov, Pisma. Zh. Eksp. Teor. Fiz. 53, 280 (1991) [JETP Lett. 53, 291 (1991)]
- A. Yu. Smirnov, Phys. Lett. B260, 161 (1991)
- E. Kh. Akhmedov, P. I. Krastev and A. Yu. Smirnov, Z. Phys. C - Particles and Fields 52, 701 (1991)
- C. Aneziris and J. Schechter, Phys. Rev. D45, 1053 (1992)
- J. Schechter, SU-4240-513 (1992)
- M. M. Guzzo and J. Bellandi, Phys. Lett. B 294, 243 (1992)
- V. M. Aquino, J. Bellandi and M. M. Guzzo, Phys. Scr. 54, 328 (1996)

Part II

2 flavor oscillations and the topological phase

Results

Detection : Split beam experiment

Two flavor oscillation formulae

Direct detection of geometric phases

- The key ingredient :

Split-beam experiment

$$||(|\psi_1\rangle + e^{i\gamma}|\psi_2\rangle)||^2 = \langle\psi_1|\psi_1\rangle + \langle\psi_2|\psi_2\rangle + e^{-i\gamma}\langle\psi_2|\psi_1\rangle + e^{i\gamma}\langle\psi_1|\psi_2\rangle$$

- A beam is split into two parts, which traverse different histories on the Poincaré sphere and finally recombined.
- Main obstacle : One needs a source and detector of neutrinos and the beam has to take two paths between them.
- The refractive index of neutrinos is so small that the focal length of any object in the solar system is astronomical. ▶ ○
- So, we cannot do what is done in optics : use mirrors or lenses to separate and recombine a beam.

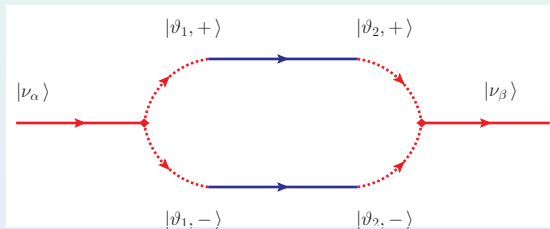
Split-beam experiment in physical space

- Clearly impossible !

Split-beam interference experiment in energy space

Think of oscillations in flavor space as

performing a split beam experiment in energy space by doing quantum collapses along with adiabatic evolution



Split-beam two-path-interferometer

Two flavor neutrino oscillation probability

Start with flavor states $|\nu_\alpha\rangle$

$$|\nu_\alpha\rangle = \nu_{\alpha+}|\vartheta_1, +\rangle + \nu_{\alpha-}|\vartheta_1, -\rangle$$

where, $|\vartheta_1, \pm\rangle$ are the eigenstates of $\mathbb{H}_\nu(\vartheta_1) = [(\sin \vartheta_1)\sigma_x + (-\cos \vartheta_1)\sigma_z]$.

Adiabatic evolution of mass states from $|\vartheta_1, \pm\rangle$ to $|\vartheta_2, \pm\rangle$

$$|\vartheta_1, \pm\rangle \rightarrow e^{-i\mathcal{D}_\pm}|\vartheta_2, \pm\rangle \quad \text{with}$$

$$\mathcal{D}_\pm \simeq \pm \frac{1}{2} \int \sqrt{(\omega \sin \vartheta)^2 + (V_C - \omega \cos \vartheta)^2} dt$$

Amplitude

$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = \langle \nu_\beta | \mathcal{U} | \nu_\alpha \rangle$ where \mathcal{U} is the unitary evolution operator given by,

$$\mathcal{U} = e^{-i\mathcal{D}_+}|\vartheta_2, +\rangle\langle\vartheta_1, +| + e^{-i\mathcal{D}_-}|\vartheta_2, -\rangle\langle\vartheta_1, -|.$$

Two flavor neutrino oscillation probability

Probability

$$\mathcal{P}(\nu_\alpha \rightarrow \nu_\beta) = |\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta)|^2$$

$$\begin{aligned} \mathcal{P}(\nu_\alpha \rightarrow \nu_\beta) = & \langle \nu_\alpha | \vartheta_1, + \rangle \langle \vartheta_2, + | \nu_\beta \rangle \langle \nu_\beta | \vartheta_2, + \rangle \langle \vartheta_1, + | \nu_\alpha \rangle \\ & + \langle \nu_\alpha | \vartheta_1, - \rangle \langle \vartheta_2, - | \nu_\beta \rangle \langle \nu_\beta | \vartheta_2, - \rangle \langle \vartheta_1, - | \nu_\alpha \rangle \\ & + [\langle \nu_\alpha | \vartheta_1, - \rangle e^{i\mathcal{D}-} \langle \vartheta_2, - | \nu_\beta \rangle \langle \nu_\beta | \vartheta_2, + \rangle e^{-i\mathcal{D}+} \langle \vartheta_1, + | \nu_\alpha \rangle + \text{c.c.}] \end{aligned}$$

- cross-terms (upon removing the dynamical phase) are connected to the two path interferometer in energy space
- can be viewed as closed loop quantum collapses with intermediate adiabatic evolutions
- great circle in $x - z$ plane.

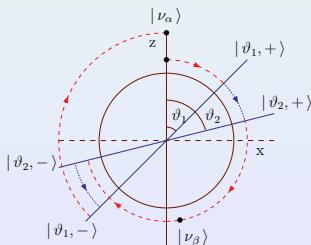
Interference terms as collapses + adiabatic evolution

Cross-terms

$$\langle \nu_\alpha | \vartheta_{1,-} \rangle \langle \vartheta_{2,-} | \nu_\beta \rangle \langle \nu_\beta | \vartheta_{2,+} \rangle \langle \vartheta_{1,+} | \nu_\alpha \rangle \equiv r e^{i\beta}$$

Appearance terms ($\nu_\alpha \rightarrow \nu_\beta$)

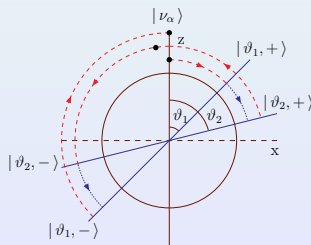
$$\beta = \pi$$



(a)

Disappearance terms ($\nu_\alpha \rightarrow \nu_\alpha$)

$$\beta = 0$$



(b)

Compare with the standard expressions

Transition probability

$$\begin{aligned}\mathcal{P}(\nu_e \rightarrow \nu_\mu) &= \mathbb{U}_{e+}^*(\theta_1) \mathbb{U}_{\mu+}(\theta_2) \mathbb{U}_{\mu+}^*(\theta_2) \mathbb{U}_{e+}(\theta_1) \\ &+ \mathbb{U}_{e-}^*(\theta_1) \mathbb{U}_{\mu-}(\theta_2) \mathbb{U}_{\mu-}^*(\theta_2) \mathbb{U}_{e-}(\theta_1) \\ &[\mathbb{U}_{e-}^*(\theta_1) e^{i\mathcal{D}-} \mathbb{U}_{\mu-}(\theta_2) \mathbb{U}_{\mu+}^*(\theta_2) e^{-i\mathcal{D}+} \mathbb{U}_{e+}(\theta_1) + \text{c.c.}]\end{aligned}$$

For the 2×2 case, $\mathbb{U}(\theta)$ is a real orthogonal rotation matrix given by,

$$\mathbb{U}(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{aligned}\mathcal{P}(\nu_e \rightarrow \nu_\mu) &= \cos^2 \theta_1 \sin^2 \theta_2 + \sin^2 \theta_1 \cos^2 \theta_2 \\ &+ [2 \cos(\mathcal{D}_+ - \mathcal{D}_-)] (-\sin \theta_1) \cos \theta_2 \sin \theta_2 \cos \theta_1\end{aligned}$$

topological phase = π

$$\begin{aligned}\mathcal{P}(\nu_e \rightarrow \nu_e) &= \cos^2 \theta_1 \cos^2 \theta_2 + \sin^2 \theta_1 \sin^2 \theta_2 \\ &+ [2 \cos(\mathcal{D}_+ - \mathcal{D}_-)] \sin \theta_1 \cos \theta_2 \sin \theta_2 \cos \theta_1\end{aligned}$$

topological phase = 0, in accord with Unitarity, $\mathcal{P}(\nu_e \rightarrow \nu_\mu) + \mathcal{P}(\nu_e \rightarrow \nu_e) = 1$

Standard expressions : vacuum and constant density matter

In vacuum for $\theta_1 = \theta_2 = \theta$

$$\mathcal{P}(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\delta m^2 l}{4E} \quad \text{and}$$

$$\mathcal{P}(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 l}{4E},$$

where in the ultra-relativistic limit, we can use $t \simeq l$ and $p \simeq E$ leading to $\mathcal{D}_\pm = \pm \delta m^2 l / 2E$ for the vacuum case ($V_C = V_N = 0$).

In matter of constant density

replace θ and δm^2 by θ^m and $(\delta m^2)^m$

Hence our result is consistent with the standard neutrino oscillation formulation and it provides a clear geometric interpretation of the phenomenon of neutrino oscillations.

Part III

Imprint of the CPV phase

Imprint of the CPV phase

Sources of CPV phase

Can CPV phases make the Pancharatnam phase geometric ?

Summary

Sources of the CPV phase

Neglecting absorption

$$\mathbb{H} = \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix} + r_0 \mathbb{I}_2 = e^{-i \int r_0 dt} \begin{bmatrix} -\cos \vartheta & \sin \vartheta e^{-i\varphi} \\ \sin \vartheta e^{i\varphi} & \cos \vartheta \end{bmatrix}$$





Medium	x	y	z
Vacuum 	$(\frac{\omega}{2}) \sin \vartheta$	0	$-(\frac{\omega}{2}) \cos \vartheta$
Ordinary medium+SI 	$(\frac{\omega}{2}) \sin \vartheta$	0	$-(\frac{\omega}{2}) \cos \vartheta + \frac{V_C}{2}$
Ordinary medium+NSI 	$\text{Re} \left((\frac{\omega}{2}) \sin \vartheta + \frac{\epsilon e y}{2} \right)$	$\text{Im} \left((\frac{\omega}{2}) \sin \vartheta + \frac{\epsilon e y}{2} \right)$	$-(\frac{\omega}{2}) \cos \vartheta + \frac{V_C}{2} + \frac{(\epsilon e e - \epsilon y y)}{2}$
Neutrino backgrounds+SI 	$\text{Re} \left((\frac{\omega}{2}) \sin \vartheta + \frac{B e y}{2} \right)$	$\text{Im} \left((\frac{\omega}{2}) \sin \vartheta + \frac{B e y}{2} \right)$	$-(\frac{\omega}{2}) \cos \vartheta + \frac{V_C}{2} + \frac{B}{2}$

Table: The three independent elements of \mathbb{H} in different kinds of media.

Pictorial depiction of the cross terms

$$\langle \psi | \vartheta_1, - \rangle \langle \vartheta_2, - | \chi \rangle \langle \chi | \vartheta_2, + \rangle \langle \vartheta_1, + | \psi \rangle \equiv r e^{i\beta}$$

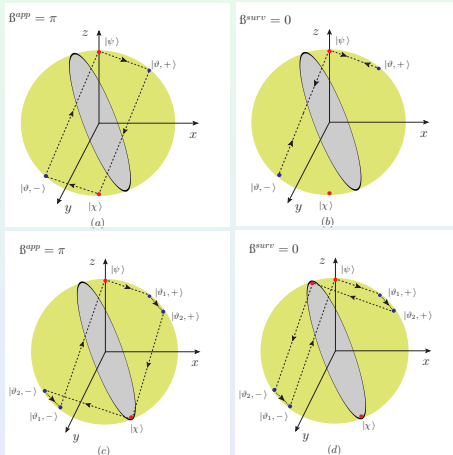


Figure: CPC situation

$$\langle \psi | \vartheta_1, \varphi_1, - \rangle \langle \vartheta_2, \varphi_2, - | \chi \rangle \langle \chi | \vartheta_2, \varphi_2, + \rangle \langle \vartheta_1, \varphi_1, + | \psi \rangle \equiv re^{iB}$$


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- This phase remains irrespective of adiabatic evolution or propagation of neutrinos in vacuum and is a robust quantity.
- The topological robustness can be destroyed once we invoke CP violation under suitable conditions.

Part IV

Extras

Characteristic scales and sensitivity to δm^2

Experiment	$L(\text{km})$	$E(\text{ GeV})$	$\delta m^2(\text{ eV}^2)$
Solar	10^7	10^{-3}	10^{-10}
Atmospheric	$10^1 - 10^4$	$10^{-1} - 10^2$	$10^{-1} - 10^{-4}$
Supernova	10^7	10^{-2}	10^{-9}
Reactor	$10^{-1} - 10^1$	10^{-3}	$10^{-2} - 10^{-3}$
Accelerator	10^{-1}	$10^{-1} - 10^1$	≥ 0.1
LBL Accelerator	$10^2 - 10^3$	10^1	$10^{-2} - 10^{-3}$

Table: Characteristic values of L and E for various neutrino experiments and sources. Note that if E is in units of MeV and L in units of m , we will obtain the same value for δm^2 that can be probed. Thus the pair (L, E) can be in the units (km, GeV) or (m, MeV) and both sets will give the same sensitivity to the value of δm^2 in eV^2 .

What about incoherent scattering effects ?

In most practical situations, the incoherent scattering cross-section of neutrinos with matter is very small \Rightarrow Sustained coherence seen even over astrophysical length scales !

Medium	ρ (g/cm ³)	$l_{mfp} = 10^{38} / (N_{Avog} \rho Y_f M E)$ (cm)
Earth core	~ 10	$\sim 10^{13}$ - 10^{19}
Solar core	~ 100	$\sim 10^{12}$ - 10^{18}
Supernova core	$\sim 10^{14}$	~ 1 - 10^6

Table: Examples of different density regions that are accessible to observations and the value of mean free path taking the target mass to be $M = 1$ GeV (1 MeV) and neutrino energy to be $E = 1$ GeV (1 MeV).

Neutrino refraction - Imaginary part

- For $\nu_e - e$ CC scattering ($E' s \ll M_W$), upto G_F^2 , using optical thm.,

$$\begin{aligned}
 \Re f(0) + i\Im f(0) &= -\frac{E}{2\pi n_e}(V_C) + i\Im f(0) \\
 &= -\frac{E}{2\pi n_e}(V_C) + i\frac{E}{4\pi}\sigma_T \\
 &= -\frac{E}{2\pi n_e}(V_C) + i\frac{E}{4\pi}\frac{1}{n_e l_{mfp}}
 \end{aligned}$$

- At zero temperature, total refractive index is

$$n_{refr} - 1 \simeq -\frac{(\sqrt{2}G_F n_e)}{E} + i\frac{1}{2E}n_e\sigma_T$$

- Real part $\Re[n_{refr}] \propto G_F$ describes coherent interference of propagating neutrinos
- Imaginary part $\Im[n_{refr}] \propto G_F^2 q^2$ is responsible for incoherent depletion of neutrinos from original coherent state
- In most situations, absorption is negligible, $\Re[n_{refr}] \gg \Im[n_{refr}]$ since neutrinos interact via weak interactions.

The π anholonomy

Non-integrable phases of ± 1 can arise in BO approximation in molecular physics (e^- spin neglected and \mathbb{H} real)

Longuet-Higgins et al., Proc. Roy. Soc. Lond. A244,1 (1958), Herzberg and Longuet-Higgins, Disc. Faraday Soc.

35, 77 (1963)

Consider a real 2×2 Hamiltonian

$$\mathbb{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix}$$

Degenerate eigenvalues \Rightarrow 2 conditions

$h_{11} - h_{22} = 0$ and $h_{12} = 0$ must be satisfied

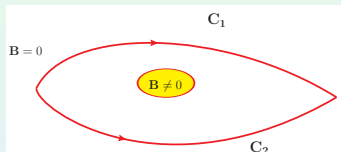
if $x = (h_{11} - h_{22})/2$ and $y = h_{12}$ then $E_{\pm} = E_0 \pm \sqrt{x^2 + y^2}$ (double cone) where $E_0 = (h_{11} + h_{22})/2$ (crossing energy)

- Encircling the degeneracy \Rightarrow eigenstate picks up a minus sign as we vary θ from $0 \rightarrow 2\pi$ continuously.
- n-times \Rightarrow phase is $(-1)^n$
- Degeneracy can be accidental or generic (need not be connected to symmetry)

The Aharonov-Bohm phase



Aharonov and Bohm, Phys. Rev. 115, 485 (1959)



- Importance of vector potential
- Even if the magnetic field $\mathbf{B} = 0$ in a certain region, the vector potential \mathbf{A} is non-zero and that causes a non-trivial phase.
- Encircling the infinite flux tube leads to anholonomy and the phase is always quantized to π .

An example : no varied parameters in \mathbb{H}

Consider a constant Hamiltonian and a general state

$$\mathbb{H} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } |\psi(0)\rangle = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 \end{pmatrix} \text{ upon evolving } |\psi(t)\rangle = \begin{pmatrix} \cos \theta/2 e^{-it/2} \\ \sin \theta/2 e^{it/2} \end{pmatrix}$$

Dynamical phase


$$\delta = \int_0^T \langle \psi | \mathbb{H} | \psi \rangle dt = T/2 (\cos^2 \theta/2 - \sin^2 \theta/2) = T/2 \cos \theta$$

Exact solution gives net phase of π for $t = 2\pi$

$$\text{because } |\psi(2\pi)\rangle = -|\psi(0)\rangle = e^{i2\pi/2} |\psi(0)\rangle$$

Missing piece is the geometric phase

$$\pi(1 - \cos \theta) = \Omega/2 \text{ where } \Omega \text{ is the solid angle subtended by a loop of fixed } \theta \text{ when } t = 2\pi \text{ i.e. } \Omega = 2\pi(1 - \cos \theta)$$

Thus, a geometric phase appears irrespective of presence of any variable parameters in Hamiltonian. 

Sun as a lens ?

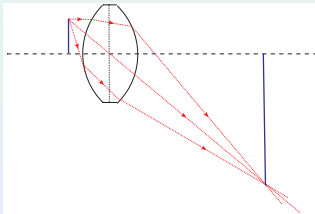
- Can we make devices similar to the optical devices using reflective and refractive property of neutrinos ?

Sun as a lens ?

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- If we take Sun as a lens, then the focal length is given by

$$f = \frac{1}{2} \frac{R_{\odot}}{(n_{refr} - 1)}$$

Lens Maker's formula (tiny n_{refr} limit)

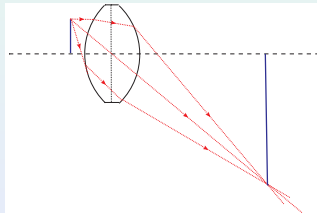


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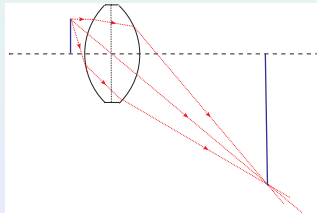
- *For 10 MeV neutrinos passing through Sun with density $\rho = 150 \text{ g cm}^{-3}$, one gets the focal length to be around $10^{18} R_{\odot} \sim 10^5$ **size of our Galaxy**.*

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- For 10 MeV neutrinos passing through Sun with density $\rho = 150 \text{ g cm}^{-3}$, one gets the focal length to be around $10^{18} R_{\odot} \sim 10^5$ size of our Galaxy.
- Potentially observable effect of small refractive index is via neutrino oscillations.

Extrinsic CPV phases

Nonstandard Interactions and their impact on coherent forward scattering

$$\begin{aligned}\mathcal{L} &= \sum_{f;\alpha,\beta} 4\frac{G_F}{\sqrt{2}} \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\beta L} (\epsilon_{\alpha\beta}^{fL} \bar{f}_L \gamma_\mu f_L + \epsilon_{\alpha\beta}^{fR} \bar{f}_R \gamma_\mu f_R) \\ \epsilon_{\alpha\beta} &= \sum_{f=e,u,d} \frac{n_f}{n_e} \epsilon_{\alpha\beta}^f\end{aligned}$$

where $f = e, p, n$ and $\alpha, \beta = e, \mu, \tau$. $\Lambda_L \gg \Lambda_{np} > \Lambda_{ew} = G_F^{-1/2}$. $\epsilon^f = \epsilon^{fL} + \epsilon^{fR}$.

Neutrino backgrounds in dense Supernovae, $|\nu_b\rangle = \gamma_e |\nu_e\rangle + \gamma_y |\nu_y\rangle$

$$\varrho_{\mathbf{p}} = |\nu_b\rangle\langle\nu_b| = \begin{bmatrix} |\gamma_e|^2 & \gamma_e \gamma_y^* \\ \gamma_e^* \gamma_y & |\gamma_y|^2 \end{bmatrix}$$

$$\begin{aligned}B &= \sqrt{2} G_F \int d^3\mathbf{q} (1 - \cos\theta_{\mathbf{p}\mathbf{q}}) \\ &\quad [(\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})_{ee} - (\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})_{yy}] \\ B_{ey} &= \sqrt{2} G_F \int d^3\mathbf{q} (1 - \cos\theta_{\mathbf{p}\mathbf{q}}) [(\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})_{ey}] \\ B_{ye} &= \sqrt{2} G_F \int d^3\mathbf{q} (1 - \cos\theta_{\mathbf{p}\mathbf{q}}) [(\varrho_{\mathbf{q}} - \bar{\varrho}_{\mathbf{q}})_{ye}]\end{aligned}$$