Topological phase in two flavor neutrino oscillations & imprint of the CPV phase

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Abstract

We show that the phase appearing in neutrino flavor oscillation formulae has a geometric and topological contribution. We identify a topological phase appearing in the two flavor neutrino oscillation formula using Pancharatnam's prescription of quantum collapses between nonorthogonal states. Such quantum collapses appear naturally in the expression for appearance and survival probabilities of neutrinos. Our analysis applies to neutrinos propagating in vacuum or through matter. For the minimal case of two flavors with CP conservation, our study shows for the first time that there is a geometric interpretation of the neutrino oscillation formulae for the detection probability of neutrino species. We also show that there is a non-trivial geometrical aspect associated with matter induced extrinsic CP violating phases when neutrinos propagate adiabatically through varying density matter. This distinction between the two cases can lead to visible consequences at the level of probability.

The Pancharatnam geometric phase

Pancharatnam (1956), Berry, (1987), Samuel and Bhandari (1988)

- Notion of geometric parallelism from inner product of two states
- Reference condition is the Pancharatnam's connection $\langle A|B\rangle$ is real and positive, in phase or parallel $|||A\rangle + |B\rangle||^2 = \langle A|A\rangle + \langle B|B\rangle + 2|\langle A|B\rangle|\cos\{ph\langle A|B\rangle\}$
- Geometrically, norm of the resultant vector is maximum
- Physically, interference of superposed beams gives maximum intensity/probability
- The connection is both symmetric and reflexive but not transitive and this fact leads to
- Pancharatnam's phase given by phase of the complex number $\langle A|C\rangle\langle C|B\rangle\langle B|A\rangle \equiv re^{i\beta}$
- β reflects curvature of the projective Hilbert space. $\beta = \frac{\Omega}{2}$ (Ω is solid angle subtended by geodesic triangle $\mathcal{ABC}$ at origin)
- Essential requirements - minimum 3 states for non-transitivity and exploring the curvature of the ray space and cyclic projections, the state need not be an eigenstate of $\mathbb{H}$.
- Schrödinger evolution possibly interrupted by measurements can lead to Pancharatnam's phase. If we take any state and subject it to multiple quantum collapses and bring it back to itself, the resulting state is

$$|A\rangle\langle A|C\rangle\langle C|B\rangle\langle B|A\rangle$$

where the phase of the complex number is half the solid angle Ω subtended by the geodesic polygon at the center of the sphere.

- In order to detect geometric phases, the key ingredient is a SPLIT BEAM EXPERIMENT

$$|||\psi_1\rangle + e^{i\beta}|\psi_2\rangle||^2 = \langle\psi_1|\psi_1\rangle + \langle\psi_2|\psi_2\rangle + e^{i\beta}\langle\psi_1|\psi_2\rangle + e^{-i\beta}\langle\psi_2|\psi_1\rangle$$

Neutrinos and some subtle points

Designing a split-beam experiment ?

- Incoherent scatterings are small in most practical situations (oscillation length being much smaller than mean free path in medium).

$$l_{osc} = \frac{\pi}{1.27}10^2 cm\frac{E}{[MeV]}\frac{[eV^2]}{\delta m^2} \quad << \quad l_{mfp} = 10^{19}cm\,\frac{[g\ cm^{-3}]}{\rho}\frac{[MeV]}{E} = 10\ light\ years$$

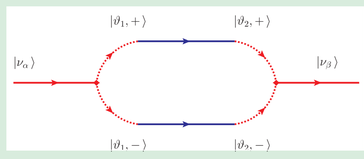
Coherence maintained over astrophysical scales.

- At zero temperature, total refractive index is

$$n_{refr} - 1 = -\frac{(\sqrt{2}G_F n_e)}{E} + i\frac{1}{2E}n_e\sigma_T \simeq -10^{-19}\frac{\rho}{[g\ cm^{-3}]}\frac{[MeV]}{E} + \ldots$$

Mostly $\Re e[n_{refr}] >> \Im m[n_{refr}]$. Its potentially observable effect occurs in neutrino oscillations which probes effects due to small mass-splittings.

- With neutrinos it is not possible to design a split beam interference experiment owing to their weakly interacting nature.
- MAIN OBSTACLE : One needs a source and detector of neutrinos and the beam has to take two paths between them.
- We can not do what is done in optics : use mirrors and lenses to separate and recombine a beam
- But, we can think of oscillations as doing a split beam experiment in energy space



Split beam two path interferometer

The π anholonomy and the form of CP-even neutrino Hamiltonian

$$\mathbb{H}_\nu = \frac{\omega}{2}[-\cos\vartheta\sigma_z + \sin\vartheta\sigma_x] \qquad |\vartheta,+\rangle = \begin{pmatrix}\cos\vartheta/2\\ \sin\vartheta/2\end{pmatrix} \quad \text{and} \quad |\vartheta,-\rangle = \begin{pmatrix}-\sin\vartheta/2\\ \cos\vartheta/2\end{pmatrix}$$

- CP conserved (coeff. of $\sigma_y = 0$), Eigenstates lie on a great circle (intersection of $x-z$ plane with Poincaré sphere)
- Eigenstates change sign as ϑ changes from $0 \to 2\pi$,

$$|\vartheta,\pm\rangle = \mp|\vartheta+\pi,\mp\rangle = -|\vartheta+2\pi,\pm\rangle$$
$$= \pm|\vartheta+3\pi,\mp\rangle = |\vartheta+4\pi,\pm\rangle$$

Global structure $\rightsquigarrow$ Möbius band

- Expect a phase of ± 1 (molecular physics) to appear in the CP-even neutrino system. Longuet-Higgins et. al. (1958)

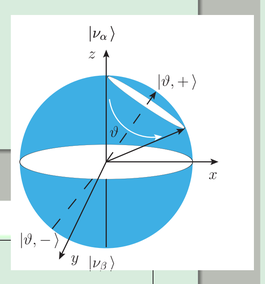
Neutrinos and Polarisation optics

2 level system and the effect of an arbitrary medium

$$\mathbb{H} = t\mathbb{I} + x\sigma_x + y\sigma_y + z\sigma_z$$

t just gives an overall phase, while x, y, z generate non-trivial optical effects.

- t and z non-zero : Circular birefringence (Optical activity)
- t and x non-zero : Linear birefringence (Wave plate)
- t and x, y, z non-zero : Elliptic birefringence (Quartz plate)
- Finite $y \implies$ CP Violation
- Absorptive effects like Dichroism : $\mathbb{H}$ need not be Hermitian



Two flavor case

Medium	x	y	z
Vacuum	$(\omega/2)\sin 2\theta$	0	$-(\omega/2)\cos 2\theta$
Normal matter+SI	$(\omega/2)\sin 2\theta$	0	$-(\omega/2)\cos 2\theta + V_C/2$
Normal matter+NSI	$(\omega/2)\sin 2\theta + \Re e\{V_C\epsilon_{e\mu}\}$	$\Im m\{V_C\epsilon_{e\mu}\}$	$-(\omega/2)\cos 2\theta + V_C/2 + V_C(\epsilon_{ee} - \epsilon_{\mu\mu})/2$
Neutrino backgrounds+SI	$(\omega/2)\sin 2\theta + \Re e\{B_{e\mu}\}$	$\Im m\{B_{e\mu}\}$	$-(\omega/2)\cos 2\theta + V_C/2 + B/2$

Visualization tool - The Poincaré sphere

$$|\psi\rangle = e^{i\eta}\begin{pmatrix}\cos(\vartheta/2)\,e^{-i\phi/2}\\ \sin(\vartheta/2)\,e^{i\phi/2}\end{pmatrix}$$

- $\mathbb{H}_\nu(\theta)$ is real ($x-z$ plane)
- Half angles used : $\vartheta = 2\theta$
- Orthogonal states - antipodal points $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle \Leftrightarrow$ RCP and LCP states
- $|\vartheta,+\rangle$ and $|\vartheta,-\rangle \Leftrightarrow$ EP states
- Oscillation phenomena can be viewed as precession, unitary rotations
- MSW effect $\Rightarrow \theta = \pi/4$ complete swapping of flavors
- NP rotated into SP (about equator with LP states at anti-podal points)
- $y \neq 0$ means $\mathbb{H}_\nu$ is complex (full Poincaré sphere)

Polarised states in optics have isomorphic connection with the neutrino states

Transition Probability

for the CP conserving case (real states)

Start with flavor states $|\nu_\alpha>$, $|\nu_\alpha> = \nu_{\alpha+}|\vartheta_1,+> + \nu_{\alpha-}|\vartheta_1,->$ where, $|\vartheta_1,\pm>$ are the eigenstates of $\mathbb{H}_\nu(\vartheta_1) = [(\sin\vartheta_1)\sigma_x + (-\cos\vartheta_1)\sigma_z]$.
Adiabatic evolution of mass states from $|\vartheta_1,\pm>$ to $|\vartheta_2,\pm>$

$$|\vartheta_1,\pm> \;\to\; e^{-i\mathcal{D}_\pm}|\vartheta_2,\pm> \quad \text{with}$$
$$\mathcal{D}_\pm \simeq \pm\frac{1}{2}\int\sqrt{(\omega\sin\vartheta)^2 + (V_C - \omega\cos\vartheta)^2}dt$$

Amplitude : $\mathcal{A}(\nu_\alpha \to \nu_\beta) = \langle\nu_\beta|\mathcal{U}|\nu_\alpha\rangle$ where

$$\mathcal{U} = e^{-i\mathcal{D}_+}|\vartheta_2,+>\langle\vartheta_1,+| + e^{-i\mathcal{D}_-}|\vartheta_2,->\langle\vartheta_1,-|\,.$$

Probability :

$$\mathcal{P}(\nu_\alpha \to \nu_\beta) = \langle\nu_\alpha|\vartheta_1,+\rangle\langle\vartheta_2,+|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,+\rangle\langle\vartheta_1,+|\nu_\alpha\rangle$$
$$+ \langle\nu_\alpha|\vartheta_1,-\rangle\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,-\rangle\langle\vartheta_1,-|\nu_\alpha\rangle$$
$$+ [\langle\nu_\alpha|\vartheta_1,-\rangle e^{i\mathcal{D}_-}\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,+\rangle e^{-i\mathcal{D}_+}\langle\vartheta_1,+|\nu_\alpha\rangle + \text{c.c.}]$$

- cross-terms (upon removing the dynamical phase) are connected to the two path interferometer in energy space
- can be viewed as closed loop quantum collapses with intermediate adiabatic evolutions
- great circle in $x-z$ plane.

Standard expression for probability

$$\mathcal{P}(\nu_e \to \nu_\mu) = \mathbb{U}^\star_{e+}(\theta_1)\mathbb{U}_{\mu+}(\theta_2)\mathbb{U}^\star_{\mu+}(\theta_2)\mathbb{U}_{e+}(\theta_1)$$
$$+ \mathbb{U}^\star_{e-}(\theta_1)\mathbb{U}_{\mu-}(\theta_2)\mathbb{U}^\star_{\mu-}(\theta_2)\mathbb{U}_{e-}(\theta_1)$$
$$+ [\mathbb{U}^\star_{e-}(\theta_1)e^{i\mathcal{D}_-}\mathbb{U}_{\mu-}(\theta_2)\mathbb{U}^\star_{\mu+}(\theta_2)e^{-i\mathcal{D}_+}\mathbb{U}_{e+}(\theta_1) + \text{c.c.}]$$

Can CP Violation destroy the topological robustness of the π phase ?

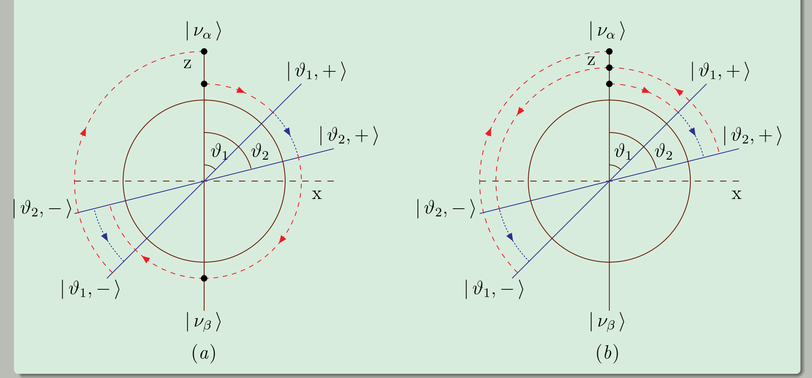
CP conserving case + adiabatic evolution

Cross-term (upon removing dynamical phase)

$\langle\nu_\alpha|\vartheta_1,-\rangle\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,+\rangle\langle\vartheta_1,+|\nu_\alpha\rangle \equiv re^{i\beta}$

Appearance terms ($\nu_\alpha \to \nu_\beta$) $\qquad$ Disappearance terms ($\nu_\alpha \to \nu_\alpha$)

$\beta = \pi \qquad\qquad \beta = 0$



$\mathcal{P}(\nu_e \to \nu_\mu) =$

$\cos^2\theta_1\sin^2\theta_2 + \sin^2\theta_1\cos^2\theta_2 + [2\cos(\mathcal{D}_+ - \mathcal{D}_-)](-\sin\theta_1)\cos\theta_2\sin\theta_2\cos\theta_1$

topological phase = $2\pi/2 = \pi$

$\mathcal{P}(\nu_e \to \nu_e) = \cos^2\theta_1\cos^2\theta_2 + \sin^2\theta_1\sin^2\theta_2 + [2\cos(\mathcal{D}_+ - \mathcal{D}_-)]\sin\theta_1\cos\theta_2\sin\theta_2\cos\theta_1$

topological phase = 0, in accord with Unitarity

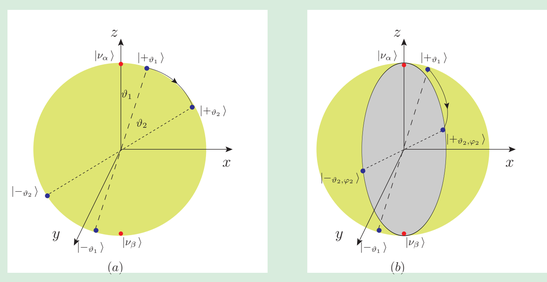
This is related to the PMNS mixing matrix being orthogonal in the 2×2 case.

CP Violating case + adiabatic evolution

Total Cross-term in the transition probability

$\mathfrak{T} = 2r\cos[\beta + (\mathcal{D}_- - \mathcal{D}_+)]$

CP Conserved ($y = 0$) $\qquad$ CP Violated ($y \neq 0$)



Appearance probability picks a phase $\Phi^{app} = (\pi - \Delta\varphi) + (\mathcal{D}_- - \mathcal{D}_+)$

geometric phase = $\Omega/2 = (\pi - \Delta\varphi)$

Survival probability picks a phase $\Phi^{surv} = \Delta\varphi + (\mathcal{D}_- - \mathcal{D}_+)$

geometric phase = $\pi - \Omega/2 = \Delta\varphi$

Essential condition to destroy topological character of the geometric phase is that $\Delta\varphi$ should be non-zero, which is realizable only in a varying density situation. It is related to lifting of states from the reference plane containing the two flavor states and the initial mass states.

Conclusions and Outlook

- We show that there exists a topological phase in the two flavor neutrino oscillation formulae by using Pancharatnam's ideas. Our study leads to first pure geometric interpretation of the phenomenon of oscillations for the specific case of two flavors and CP conserving case.

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- The non-trivial phase of π and the anholonomy is linked to encircling of a singular point in ray space.
- We made a direct connection to the π phase anholonomy first found in the context of molecular physics by Longuet-Higgins et. al. in 1958.
- The phase remains irrespective of adiabatic evolution or propagation of neutrinos in vacuum and is a robust quantity.
- It is in-built into the structure of the leptonic mixing matrix. For the 2×2 case and CP conservation, $\mathbb{U}(\theta)$ is a real orthogonal rotation matrix given by,

$$\mathbb{U}(\theta) = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

Therefore the standard formalism of oscillation is in fact a realization of the Pancharatnam phase.

- The topological robustness can be destroyed once we invoke CP violation. Thus our studies with geometric phases lead to a novel quantification of effects due to CP violating phase present in the Hamiltonian which are very hard to visualize otherwise.

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- In presence of CP violation, the mixing matrix can be made orthogonal initially but the final mixing matrix takes the following form

$$\mathbb{U}(\theta_2) = \begin{pmatrix}\cos\theta_2 & \sin\theta_2\\ -\sin\theta_2 & \cos\theta_2\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{i\Delta\varphi}\end{pmatrix}$$

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