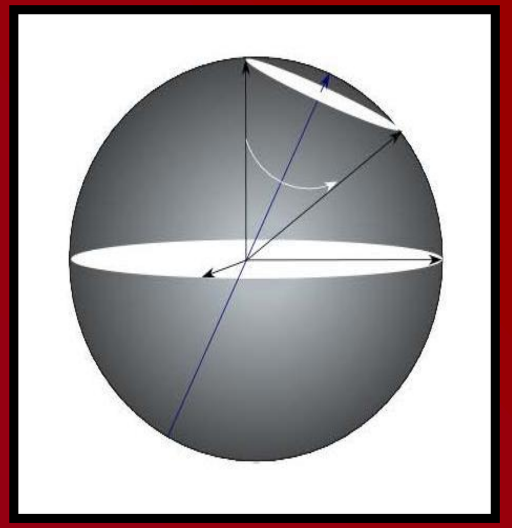

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Topological phase in two flavor neutrino oscillations

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2-flavor neutrinos and polarisation optics

2 neutrino flavor states in the ultra-relativistic limit

Can be mapped to a 2 level quantum system with energy eigenvalues as

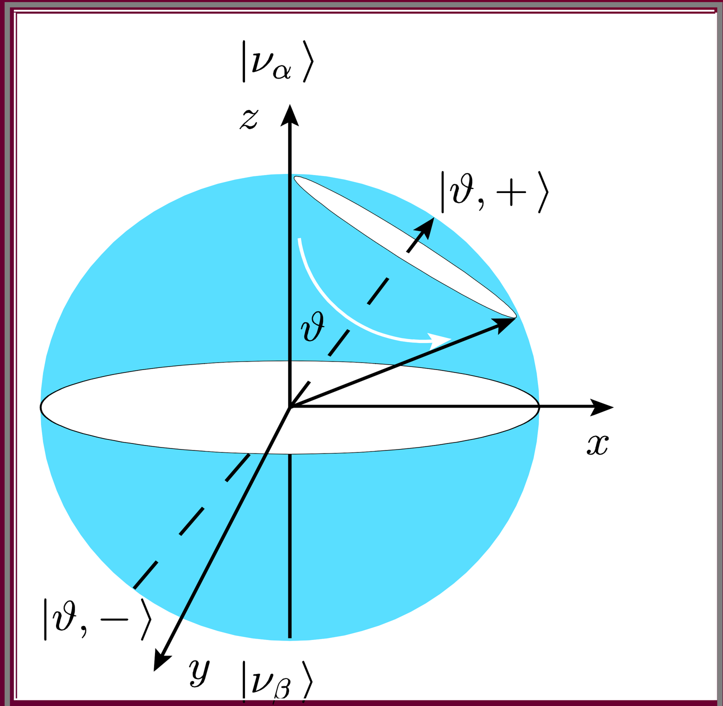
$$E_1 = \sqrt{p^2 + m_1^2} \approx p + \frac{m_1^2}{2p}; \quad E_2 = \sqrt{p^2 + m_2^2} \approx p + \frac{m_2^2}{2p}$$

in natural units

for fixed p and x~t

Neutrinos are produced and detected via weak interactions:
weak eigenstates differ from mass eigenstates

This leads to oscillation which is similar to birefringence in optics



Hamiltonian is real
Half angles used
Orthogonal states are antipodal points
Flavor states – RCP and LCP states
Mass states – EP states
Oscillation phenomena viewed as precession or Unitary rotations
MSW effect – NP into SP, complete swap of flavors

Effect of medium can be described in terms of

$$\mathbf{H} = D\mathbf{I} + A\sigma_1 + B\sigma_2 + C\sigma_3$$

D → just gives an overall phase
while A, B and C generate non-trivial optical effects
Circular Birefringence (Optical activity) *– D and C non-zero and A=B=0*
Linear Birefringence (Wave plate) *– D and A non-zero and B=C=0*
Elliptic Birefringence (Quartz plate) *– D, A, B, C non-zero* ***(most general)***
Absorption effects like DICHROISM neglected since the incoherent scattering cross-section for neutrinos is extremely small in comparison to photons in a medium.

In ordinary matter, ***Elliptic Birefringence***

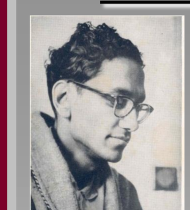
$$\mathbf{H} = D\mathbf{I} + \frac{1}{2}\begin{pmatrix} -\omega\cos 2\theta + V_C & \omega \sin 2\theta \\ \omega\sin 2\theta & \omega\cos 2\theta - V_C \end{pmatrix}$$

Standard Model interactions with any number of constituents – net effect is that only one parameter in the Hamiltonian that depends on time.

Most dramatic effect is the MSW resonance due to vanishing off-diagonal terms
Absence of FCNC leads to the fact that matter by itself cannot induce oscillations

The Pancharatnam Phase

Phase between non-orthogonal states



Pancharatnam (1956), Berry (1987), Samuel and Bhandari (1988)

Notion of geometric parallelism from inner product of two states

Reference condition: ***Pancharatnam's connection***

$\langle A|B\rangle$ is real and positive, in phase or parallel

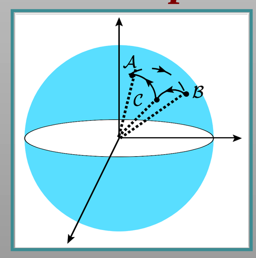
$$\big\| |A\rangle + |B\rangle \big\|^2 = \langle A|A\rangle + \langle B|B\rangle + 2|\langle A|B\rangle| \cos(ph\langle A|B\rangle)$$

Geometrically, norm of resultant vector is maximum.
Physically, interference of superposed beams gives maximum probability/intensity

Pancharatnam's connection is both reflexive and symmetric but not transitive → Pancharatnam's phase

phase of the complex number

$$\langle A|C\rangle\langle C|B\rangle\langle B|A\rangle \equiv \mathrm{r}\exp(i\beta) = \frac{\Omega}{2}$$



β reflects curvature of the projective Hilbert space.
Essential requirements – minimum 3 states for non-transitivity and exploring the curvature of the ray space and cyclic projections , the state need not be an eigenstate of H.

Schrodinger evolution possibly interrupted by measurements can lead to Pancharatnam's phase
If we take any state and subject it to multiple quantum collapses and bring it back to itself, the resulting state

$$|A\rangle\langle A|C\rangle\langle C|B\rangle\langle B|A\rangle$$

where the phase of the complex number is half the solid angle subtended by the geodesic polygon at the center of the sphere.

The key ingredient is the split beam experiment

$$\big\| |\psi_1\rangle + \exp(i\gamma)|\psi_2\rangle \big\|^2 = \langle\psi_1|\psi_1\rangle + \langle\psi_2|\psi_2\rangle + \exp(-i\gamma)\langle\psi_2|\psi_1\rangle + \exp(i\gamma)\langle\psi_1|\psi_2\rangle$$

With neutrinos it is not possible to design a split beam interference experiment owing to their weakly interacting nature…

Tiny refractive index =>
Its potentially observable effect occurs in neutrino oscillations which probes effects due to small mass-splittings

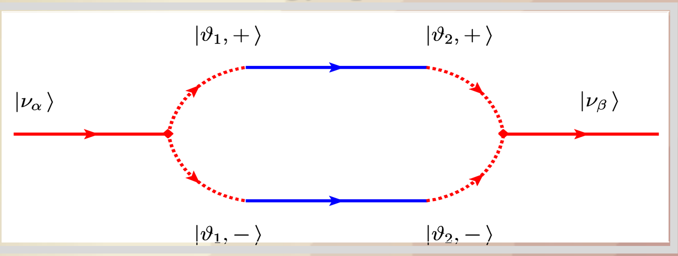
Incoherent scatterings are small in most practical situations (oscillation length being much smaller than mean free path in medium).
Coherence maintained over astrophysical scales.

$$l_{\mathrm{mfp}} = 10^{19}\,\mathrm{cm}\,\frac{1\,\mathrm{g/cc}}{\rho}\,\frac{1\,\mathrm{MeV}}{\mathrm{E}} \approx 10\ \text{light years}$$

$$n_{\mathrm{refr}} - 1 = 10^{-19}\,\frac{\rho}{1\,\mathrm{g/cc}}\,\frac{1\,\mathrm{MeV}}{\mathrm{E}}$$

$$l_{\mathrm{osc}} = \frac{\pi}{1.27}\,10^2\,\mathrm{cm}\,\frac{\mathrm{E}}{1\,\mathrm{MeV}}\,\frac{1\,\mathrm{eV}^2}{\delta m^2}$$

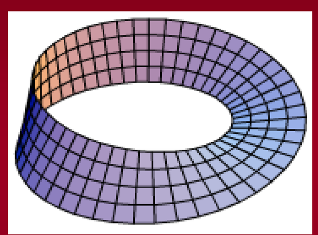
Think of oscillations as doing a split-beam experiment in energy space



by doing collapses and intermediate adiabatic evolutions

$$\mathrm{H} = \frac{\omega}{2}\left[-\cos\vartheta\,\sigma_z + \sin\vartheta\,\sigma_x\right]$$

$$|\vartheta,+\rangle = \begin{pmatrix}\cos\vartheta/2 \\ \sin\vartheta/2\end{pmatrix}; \quad |\vartheta,+\rangle = \begin{pmatrix}-\sin\vartheta/2 \\ \cos\vartheta/2\end{pmatrix}$$



$$|\vartheta,\pm\rangle = \mp|\vartheta+\pi,\mp\rangle = -|\vartheta+2\pi,\pm\rangle = \pm|\vartheta+3\pi,\mp\rangle = |\vartheta+4\pi,\pm\rangle$$

Examine the form of Hamiltonian for two flavor neutrino system
CP conserved, eigenstates lie on a great circle
Eigenstates change sign as we change the angle from 0 to 2 pi.
Expect the pi phase that was first found in molecular physics to also appear in the neutrino system

Longuet-Higgins et. al. (1958)

Transition Probability and the Pi phase

Start with the flavor state $|\nu_\alpha\rangle$

$$|\nu_\alpha\rangle = \nu_{\alpha+}|\vartheta_1,+\rangle + \nu_{\alpha-}|\vartheta_1,-\rangle$$

$|\vartheta_1,\pm\rangle$ are the eigenstates of the Hamiltonian

$$H_\nu(\vartheta_1) = [(\sin\vartheta_1)\sigma_x + (-\cos\vartheta_1)\sigma_z]$$

Adiabatic evolution of mass states

$$|\vartheta_1,\pm\rangle \rightarrow e^{-iD_\pm}|\vartheta_2,\pm\rangle$$

$$D_\pm \approx \pm\frac{1}{2}\int\sqrt{(\omega\sin\vartheta)^2 + (V_C - \omega\sin\vartheta)^2}\,dt$$

Amplitude $\mathrm{A}(\nu_\alpha \rightarrow \nu_\beta) = \langle\nu_\beta|\mathrm{U}|\nu_\alpha\rangle$

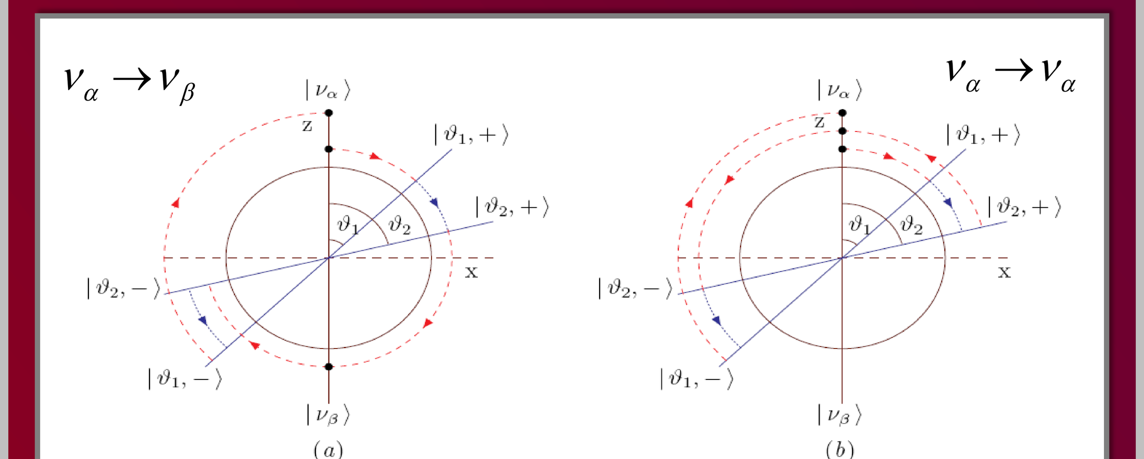
$$\mathrm{U} = e^{-iD_+}|\vartheta_2,+\rangle\langle\vartheta_1,+| + e^{-iD_-}|\vartheta_2,-\rangle\langle\vartheta_1,-|$$

Probability $\mathrm{P}(\nu_\alpha \rightarrow \nu_\beta) = |\mathrm{A}(\nu_\alpha \rightarrow \nu_\beta)|^2$

$$\begin{aligned}\mathrm{P}(\nu_\alpha \rightarrow \nu_\beta) = {} & \langle\nu_\alpha|\vartheta_1,+\rangle\langle\vartheta_2,+|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,+\rangle\langle\vartheta_1,+|\nu_\alpha\rangle + \\ & \langle\nu_\alpha|\vartheta_1,-\rangle\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,-\rangle\langle\vartheta_1,-|\nu_\alpha\rangle + \\ & \left[\langle\nu_\alpha|\vartheta_1,-\rangle e^{iD_-}\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,-\rangle e^{-iD_+}\langle\vartheta_1,+|\nu_\alpha\rangle + \mathrm{c.c}\right]\end{aligned}$$

The cross term in probability is

$$\langle\nu_\alpha|\vartheta_1,-\rangle e^{iD_-}\langle\vartheta_2,-|\nu_\beta\rangle\langle\nu_\beta|\vartheta_2,-\rangle e^{-iD_+}\langle\vartheta_1,+|\nu_\alpha\rangle \equiv \mathrm{r}\,\mathrm{e}^{\mathrm{i}\beta}$$



Appearance probability

$$\begin{aligned}\mathrm{P}(\nu_e \rightarrow \nu_\mu) = {} & \cos^2\theta_1\sin^2\theta_2 + \sin^2\theta_1\cos^2\theta_2 + \\ & [2\cos(D_+ - D_-)](-\sin\theta_1)\cos\theta_2\sin\theta_2\cos\theta_1\end{aligned}$$

Survival probability

$$\begin{aligned}\mathrm{P}(\nu_e \rightarrow \nu_e) = {} & \cos^2\theta_1\cos^2\theta_2 + \sin^2\theta_1\sin^2\theta_2 + \\ & [2\cos(D_+ - D_-)]\sin\theta_1\cos\theta_2\sin\theta_2\cos\theta_1\end{aligned}$$

Conclusions

We show that there is a topological phase in the two flavor neutrino oscillation formulae by using Pancharatnam's ideas and our study leads to first pure geometric interpretation of the phenomenon of oscillations for the specific case of two flavors and CP conserving case.

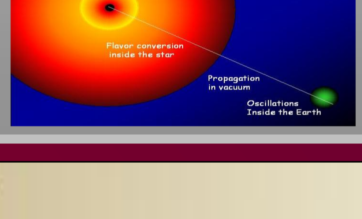
The non-trivial phase of pi and the anholonomy is linked to encircling of a singular point in ray space.

We made a direct connection to the pi phase anholonomy first found in the context of molecular physics by Longuet-Higgins et. al. in 1958.

The phase remains irrespective of adiabatic evolution or propagation of neutrinos in vacuum and is a robust quantity.

It is in-built into the structure of the PMNS mixing matrix and hence the standard formalism of oscillation is in fact a realization of the Pancharatnam phase..

The topological robustness can be destroyed once we invoke CP violation.
Mehta (2009), 0907.0562 [hep-ph]



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