**2-flavor neutrinos and polarisation optics**

**Phase between non-orthogonal states:**

Pancharatnam (1968, Berry (1987), Samuel and Bhandari (1988))

Notion of geometric parallelism from inner product of two states

Reference condition: Pancharatnam's connection

\[ \langle A | B \rangle = \langle A | + B \rangle \]

Geometrically, norm of resultant vector is maximum.

Physically, interference of superposed beams gives maximum probability/intensity

Pancharatnam's connection is both reflective and symmetric but not transitive \( \rightarrow \) Pancharatnam's phase

\[ \beta \rightarrow \] reflects curvature of the projective Hilbert space.

Essential requirements – minimum 3 states for non-transitivity and exploring the curvature of the ray space and cyclic projections, the state need not be an eigenstate of \( H \).

**The Pancharatnam Phase**

Schrödinger evolution possibly interrupted by measurements can lead to Pancharatnam’s phase. If we take any state and subject it to multiple quantum collapses and bring it back to itself, the resulting state

\[ \langle A | C \rangle \langle C | B \rangle \langle B | A \rangle \]

where the phase of the complex number is half the solid angle subtended by the geodesic polygon at the center of the sphere.

The key ingredient is the split beam experiment

\[ \langle \psi_{\alpha} \rangle = \langle \psi_{\alpha} | \psi_{\alpha} \rangle \]

With neutrinos it is not possible to design a split beam interference experiment owing to their weakly interacting nature.

Incoherent scatterings are small in most practical situations (oscillation length being much smaller than mean free path in medium).

Coherence maintained over astrophysical scales.

**Transition Probability and the Pi phase**

The cross term in probability is

\[ \langle v_\alpha | \delta \rangle \langle \delta | v_\alpha \rangle \]

Appearance probability

\[ P(v_\alpha \rightarrow v_{\beta}) = \cos^2 \theta_{12} \sin^2 \theta_{13} \cos \theta_{23} + \sin^2 \theta_{13} \cos \theta_{23} + \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} + \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} \]

Survival probability

\[ P(v_\alpha \rightarrow v_\alpha) = \cos^2 \theta_{12} \cos^2 \theta_{13} + \sin^2 \theta_{13} \sin^2 \theta_{23} + \sin \theta_{13} \cos \theta_{13} \sin \theta_{23} \cos \theta_{23} \]

**Conclusions**

We show that there is a topological phase in the two flavor neutrino oscillation formulae by using Pancharatnam’s ideas and our study leads to first pure geometric interpretation of the phenomenon of oscillations for the specific case of two flavors and CP conserving case.

The non-trivial phase of \( \pi \) and the anholonomy is linked to encoding of a singular point in ray space.

We made a direct connection to the pi phase anholonomy first found in the context of molecular physics by Longuet-Higgins et al. in 1958.

The phase remains irrespective of adiabatic evolution or propagation of neutrinos in vacuum and is a robust quantity.

It is in-built into the structure of the PMNS mixing matrix and hence the standard formalism of oscillation is in fact a realization of the Pancharatnam phase.

The topological robustness can be destroyed once we invoke CP violation.

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