



# Nonlocal Pancharatnam Phase in Two-Photon Interferometry

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## Abstract

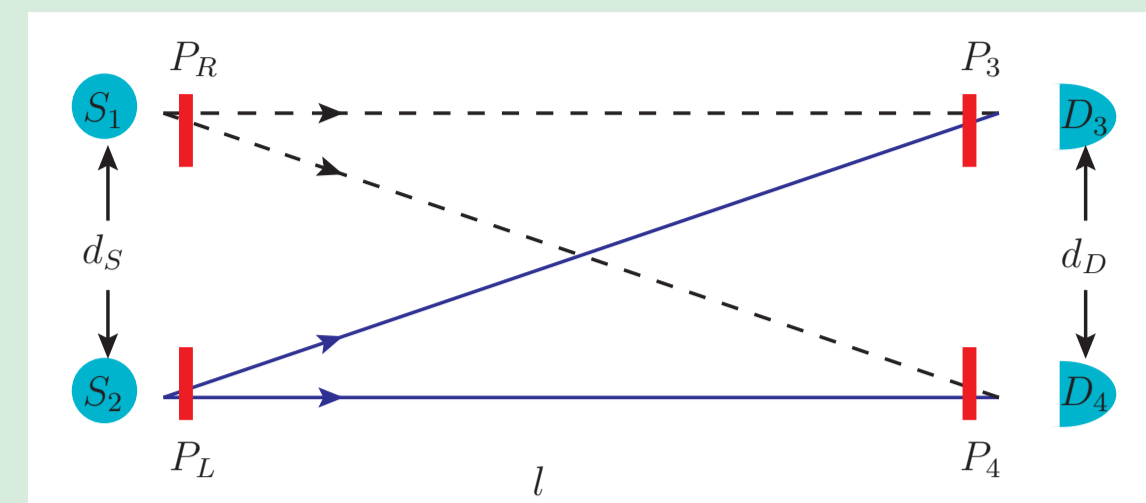
We propose a polarised intensity interferometry experiment, which measures the nonlocal Pancharatnam phase acquired by a pair of Hanbury Brown-Twiss (HB-T) photons. The setup involves two polarised thermal sources illuminating two polarised detectors. Varying the relative polarisation angle of the detectors introduces a two photon geometric phase. Local measurements at either detector do not reveal the effects of the phase, which is an optical analog of the multiparticle Aharonov-Bohm effect. The geometric phase sheds light on the three slit experiment and suggests ways of tuning entanglement.

## Historic and Conceptual Background

- While Quantum Mechanics was born in the nineteen twenties, many significant conceptual and foundational developments which stem from Quantum Mechanics emerged much later. The multiparticle effects in intensity interferometry were only understood in the nineteen fifties, the Aharonov-Bohm effect in the sixties, entanglement, nonlocal correlations and Bell's inequalities were appreciated in the seventies and developments related to the geometric phase emerged in the eighties.
- The present poster weaves together these several conceptual strands into a single experimental proposal. It is an optical analogue of the multiparticle nonlocal Aharonov-Bohm experiment which was reported by Neder et al., Nature, 448, 333, 2007 in the context of the Quantum Hall effect.
- In our proposed optics experiment, the arrival times of photons at two detectors show correlations (coincidence rates) which are sensitive to a nonlocal, multiparticle geometric phase.
- It leads to the possibility of tuning the orbital entanglement of photons using this geometric phase. Interest in intensity interferometry spans over different branches of physics ranging across nuclear and particle physics, astrophysics, optics and condensed matter physics.
- It was only with the HB-T interferometer that the possibility of multiparticle correlations was appreciated. Similarly Berry's discovery of the geometric phase led to deeper understanding of wide range of phenomena in different areas of physics starting from molecular physics to Quantum Field Theory.
- In the present work, we combine our understanding of the Pancharatnam's geometric phase with that of intensity interferometry to obtain a conceptually new result in optics.

## Proposed Experiment

### Slight modification of the HB-T Experiment



- The experiment consists of having two thermal sources  $S_1$  and  $S_2$  illuminate two detectors  $D_3$  and  $D_4$ . This setup is very similar to the HB-T experiment. The only difference is in the use of analysers, which select a particular state of polarisation.
- The source  $S_1$  is covered by an analyser  $P_R$ , which only permits Right Hand Circular light to pass through it, while the source  $S_2$  is covered by an analyser  $P_L$ , which only permits Left Hand Circular light to pass through.
- The light is incident on detectors  $D_3$  and  $D_4$  after passing through polaroids  $P_3$  and  $P_4$  respectively that only permit linearly polarised light to pass through (linear analysers).
- The angle  $\varphi_{34}$  between the axes of  $P_3$  and  $P_4$  and the detector separation  $d_D$  can be continuously varied in the experiment.
- The measured quantity is the coincidence count  $\mathcal{C}$  of photons received at detectors  $D_3$  and  $D_4$ ,

$$\mathcal{C} = \mathcal{G}^2 = \frac{\langle N_3 N_4 \rangle}{\langle N_3 \rangle \langle N_4 \rangle}, \quad (1)$$

where  $N_3$  and  $N_4$  are the photon numbers detected at  $D_3$  and  $D_4$  per unit time (per unit bandwidth).

- As in the HB-T interferometer, we would expect the coincidence counts to vary with the propagation phases and so the counts would depend on the detector separation  $d_D$  and the wavelength  $\lambda$  of the light. The new effect that is present in the polarised version is that we expect the coincidence counts to also depend on  $\varphi_{34}$  and to be modulated by a geometric phase of half the solid angle on the Poincaré sphere. The geometric phase is achromatic, unlike the propagation phases mentioned above.
- Note that the path traversed on the Poincaré sphere is not traced by a single photon, but by a pair of HB-T photons. Thus the experiment explores a new avatar of the geometric phase in the context of intensity interferometry.

## Theory - Coincidence Counts and the Geometric Phase

### Notation and simplifying assumptions

- quasi-monochromatic beam.
- $I \gg d_S, d_D$ .
- $a_1^\alpha$  and  $a_2^\alpha$ : the destruction operators of the photon modes at the sources  $S_1$  and  $S_2$  where  $\alpha$  runs over the two polarisation states.
- $a_3^\alpha$  and  $a_4^\alpha$ : the destruction operators of the photon modes at detectors
- Modes just after the analysers:  $a_R^\alpha = P_R^{\alpha\beta} a_1^\beta$  and  $a_L^\alpha = P_L^{\alpha\beta} a_2^\beta$  where a sum over repeated Greek indices is understood and the projection matrices  $P_R$  and  $P_L$  onto the right and left circular states represent the action of the analysers.
- Propagation phases

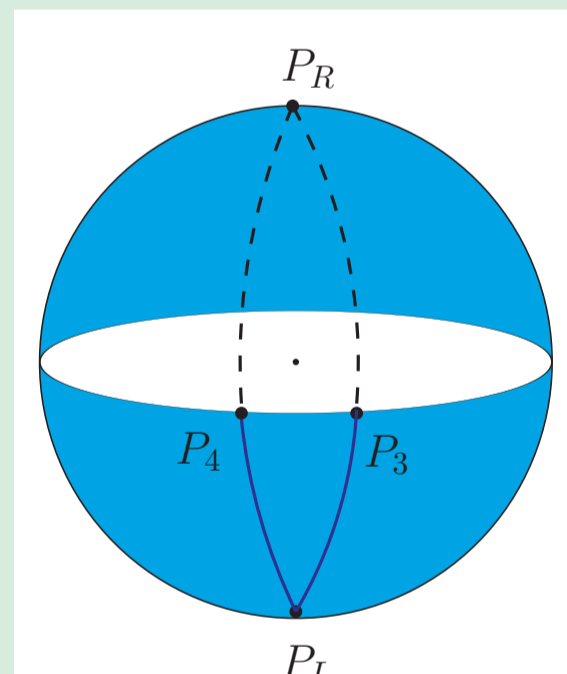
$$u_{ij} = \frac{1}{I} \exp \{ i[k(|\vec{r}_i - \vec{r}_j|) - \omega t] \},$$

where  $\omega$  is the frequency of the light,  $k$  is the wave vector and  $\vec{r}_i$  and  $\vec{r}_j$  the locations of the detector and source.

- $a_b^\alpha$  ( $b = 3, 4$ ) and its Hermitean conjugate  $a_b^{\dagger\alpha}$  are then

$$\begin{aligned} a_b^\alpha &= P_b^{\alpha\beta} [P_L^{\beta\gamma} a_2^\gamma u_{b2} + P_R^{\beta\gamma} a_1^\gamma u_{b1}] \\ a_b^{\dagger\alpha} &= [\bar{u}_{b2} a_2^{\dagger\gamma} P_L^{\gamma\beta} + \bar{u}_{b1} a_1^{\dagger\gamma} P_R^{\gamma\beta}] P_b^{\beta\alpha}, \end{aligned}$$

where the overbar stands for complex conjugation and we use the fact that the  $2 \times 2$  Hermitean projection matrices  $P$  satisfy  $P^2 = P$  and  $\bar{P}^{\alpha\beta} = P^{\beta\alpha}$ .



### Quantities of interest: $\langle N_3 \rangle$ , $\langle N_4 \rangle$ and $\langle : N_3 N_4 : \rangle$

- $N_b = a_b^{\dagger\alpha} a_b^\alpha = [\bar{u}_{b2} a_2^{\dagger\alpha} (P_L P_b)^{\alpha\beta} + \bar{u}_{b1} a_1^{\dagger\alpha} (P_R P_b)^{\alpha\beta}] [P_L^{\beta\gamma} a_2^\gamma u_{b2} + P_R^{\beta\gamma} a_1^\gamma u_{b1}] = \bar{u}_{b1} u_{b1} (P_R P_b P_R)^{\alpha\beta} \langle a_1^{\dagger\alpha} a_1^\beta \rangle + \bar{u}_{b2} u_{b2} (P_L P_b P_L)^{\alpha\beta} \langle a_2^{\dagger\alpha} a_2^\beta \rangle$  From the thermal nature of the sources,  $\langle a_1^{\dagger\alpha} a_1^\beta \rangle = \langle a_2^{\dagger\alpha} a_2^\beta \rangle = \delta^{\alpha\beta} n_B$  where  $n_B$  is the Bose function  $(\exp\{\beta\hbar\omega\} - 1)^{-1}$  and  $\beta$  the inverse temperature. We obtain

$$\langle N_3 \rangle = \langle N_4 \rangle = \frac{n_B}{\bar{I}^2} \quad (2)$$

- The computation of  $\langle : N_3 N_4 : \rangle$  is slightly more involved but straightforward. The product  $N_3 N_4$  is a product of four brackets each of which has two terms. When the four brackets are expanded, there are sixteen terms, of which ten vanish. The six nonzero terms combine to give

$$\langle : N_3 N_4 : \rangle = n_B^2 \left[ \frac{3}{2\bar{I}^4} + \bar{u}_{32} u_{31} \bar{u}_{41} u_{42} \text{Tr}[P_L P_3 P_R P_4 P_L] + \bar{u}_{31} u_{32} \bar{u}_{42} u_{41} \text{Tr}[P_R P_3 P_L P_4 P_R] \right]. \quad (3)$$

Only the second and third terms in Eq. (3) contain the propagation and geometric phases.

- The sequence of projections can be viewed as a series of closed loop quantum collapses given by  $\langle R | 3 \rangle \langle 3 | L \rangle \langle L | 4 \rangle \langle 4 | R \rangle$

$$\text{Tr}[P_R P_3 P_L P_4 P_R] = \frac{1}{4} \exp \left\{ i \frac{\Omega}{2} \right\}, \quad (4)$$

where  $\Omega$  is the solid angle subtended by the geodesic path  $|R\rangle \rightarrow |3\rangle \rightarrow |L\rangle \rightarrow |4\rangle \rightarrow |R\rangle$  at the center of the Poincaré sphere. Apart from the phase, the projections also result in an amplitude factor of  $1/4$  since projections are non-unitary operations leading to a loss in intensity.

- The final theoretical expression for  $\mathcal{C}$  in the limit  $I \gg d_S, d_D$  is

$$\mathcal{C} = \frac{3}{2} + \frac{1}{2} \cos \left[ \vec{d}_D \cdot (\vec{k}_2 - \vec{k}_1) + \frac{\Omega}{2} \right], \quad (5)$$

where  $\vec{k}_i = k\hat{r}_i$  is the wavevector of light seen in the  $i^{\text{th}}$  detector. (The propagation phases in Eq. (5) can also be written in an equivalent form with the sources and detectors exchanged.).

- It is also easily seen that the self correlation  $\langle : N_3 N_3 : \rangle$  ( $\langle : N_4 N_4 : \rangle$ ) can be obtained by replacing  $4$  by  $3$  ( $3$  by  $4$ ) in Eq. (3) above. In this case, the sequence of projections  $\text{Tr}[P_R P_3 P_L P_3 P_R]$  ( $\text{Tr}[P_R P_4 P_L P_4 P_R]$ ) subtends a zero solid angle and the geometric contribution to the phase vanishes. Thus neither the photon counts  $\langle N_3 \rangle$ ,  $\langle N_4 \rangle$  in individual detectors nor the self correlations  $\langle : N_3 N_3 : \rangle$ ,  $\langle : N_4 N_4 : \rangle$  reveal the geometric phase. This supports our claim that the effect described here is only present in the cross-correlations and not in the self correlations.
- $\mathcal{C}$  depends on the experimentally tunable parameters  $d_D$  and  $\varphi_{34}$ . The geometric part is achromatic and depends only on  $\varphi_{34}$ . The propagation part in the phase carries the dependence on  $d_D$  as well as on the wavelength. By changing the angle  $\varphi_{34}$  between the axes of the two polaroids, we can conveniently modulate the geometric component  $\Omega$ . If the propagation and geometric phases are set to zero, we find that the correlation  $\mathcal{C}$  takes the value  $2$ , just as in original HB-T interferometry.

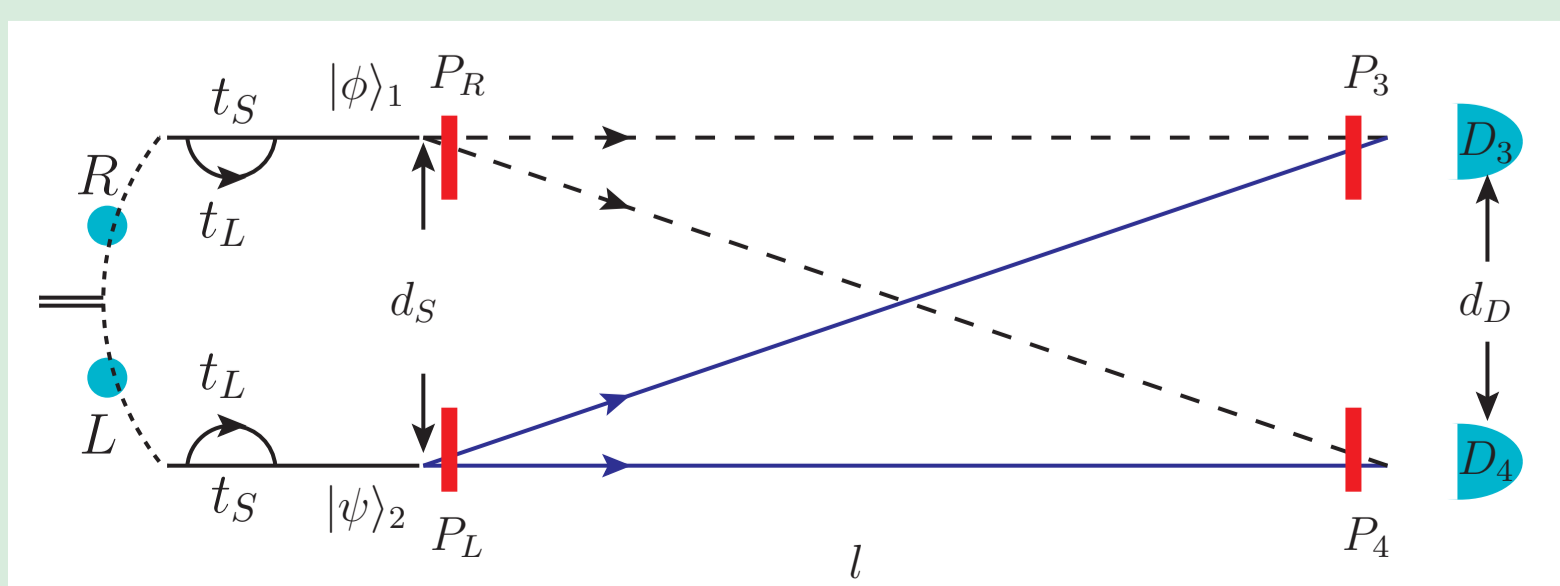
## Conclusion and Further Implications

### Final Remarks

- We have proposed a simple generalisation of the HB-T experiment [Hanbury-Brown and Twiss, Nature 177, 27 (1956)] which uses the vector nature of light to produce a geometric phase.
- The only difference between the proposed experiment and the HB-T experiment is the presence of polarisers at the sources and detectors. These polarisers cause a geometric phase to appear in the coincidence counts of two detectors which receive linearly polarised light.
- Neither the count rates nor the self correlations of individual detectors show any geometric phase effects. These appear solely in the cross correlations in the count rates of the detectors.
- The appearance of the geometric phase cannot be attributed or localised to any single segment joining a source ( $S_1, S_2$ ) to a detector ( $D_3, D_4$ ). It appears only when one considers the two photon path on the Poincaré sphere in its entirety.
- Our experiment brings out a new result of a conceptual nature, which may not have been guessed without our present understanding of the Pancharatnam phase. The experiment proposed here would be a good demonstration of a purely multiparticle and nonlocal geometric phase in optics. We hope to interest experimentalists in this endeavour.
- The ideas can be extended to other particles than photons such as neutrons, electrons or neutrinos.
- Apart from verifying the theoretical expectation, our proposed experiment suggests further lines of thought concerning multiparticle and nonlocal effects which may be stimulating to research in this area. We mention two of these, the first an application of our ideas to generating controlled entanglement and the second of a more conceptual nature regarding the role of probabilities in quantum mechanics.

### Controlled entanglement via particle exchange

- Like many other elementary particles, the photon has spin (polarisation) as well as orbital (spacetime) degrees of freedom. Our idea is to use the polarisation degree of freedom to control the orbital entanglement of photons. Let us replace the two thermal sources by a single two-photon source producing a pair of oppositely circularly polarised photons.
- Each photon is then passed through an interferometric delay line which consists of a short and a long arm with time delays  $t_S$  and  $t_L$ .
- The relative amplitudes and phases of the two paths can be chosen to generate any state in the two dimensional Hilbert space spanned by  $|S\rangle$  and  $|L\rangle$ . By such means we can arrange for the incident state at  $P_R$  to be in a spin state of right circular polarisation and in an orbital state  $|\phi\rangle_1 = \alpha|S\rangle_1 + \beta|L\rangle_1$  and similarly, the incident state at  $P_L$  to be in a spin state of left circular polarisation and in an orbital state  $|\psi\rangle_2 = \alpha'|S\rangle_2 + \beta'|L\rangle_2$ , where  $\alpha, \beta$  etc are complex numbers.
- The input state is therefore a direct product of states at  $P_R$  and  $P_L$ :  $|\phi\rangle_1 \otimes |\psi\rangle_2$ . By combining the amplitudes for the two photons to arrive at the detectors via the paths  $1 - 3, 2 - 4$  and  $1 - 4, 2 - 3$  (direct and exchange) we find that the state at the output is of the form  $|\phi\rangle_3 \otimes |\psi\rangle_4 + \exp\{i\Omega/2\} |\psi\rangle_3 \otimes |\phi\rangle_4$ , where the geometric phase factor  $\exp\{i\Omega/2\}$  is the relative phase between the direct and exchange processes.
- This final two photon state is entangled as it cannot in general be written as a direct product  $|\Psi\rangle_3 \otimes |\Phi\rangle_4$  of photon states at  $3$  and  $4$ .
- The entanglement is generated by particle exchange effects rather than interactions.



### Three slit experiment

- Two slit experiment: The outcome of the two-slit experiment is not determined by the outcome of one-slit experiments in which one or the other of the slits is blocked. This is in sharp contrast to classical random processes like Brownian motion. Thus classical probabilities are one slit separable, but quantum probabilities are not!
- Three slit experiment: If we consider three slits  $A, B, C$ , we find that in quantum mechanics, the outcome of the three-slit experiment is determined by the outcomes of the one and two slit experiments i.e.,  $\mathcal{P}_{ABC} = \mathcal{P}_{AB} + \mathcal{P}_{BC} + \mathcal{P}_{CA} - \mathcal{P}_A - \mathcal{P}_B - \mathcal{P}_C$ , which follows easily from writing  $\mathcal{P}_{ABC} = |\psi_A + \psi_B + \psi_C|^2$  and  $\psi_A, \psi_B, \psi_C$  are the amplitudes for passage through the slits. Thus quantum mechanics is two slit separable. This is why we do not find a discussion of the three slit experiment in elementary Quantum Mechanics books: it brings in nothing new.
- Three slit experiment + multiparticle and nonlocal processes: Consider a three-slit experiment in which three incoherent beams of light fall upon three slits  $A, B, C$  which are covered by analysers  $P_A, P_B, P_C$  each of which allows a single state on the Poincaré sphere to pass. The light from the analysers is then allowed to fall on three unpolarised detectors labelled  $4, 5, 6$ . We find that the number correlations  $\langle N_4 N_5 N_6 \rangle$  contain terms involving the geometric phase (half the solid angle subtended by the three polarisation states  $A, B, C$  of the analysers). Such an effect is not present in any of the two-slit or one-slit experiments, since two (or fewer) points on the Poincaré sphere do not enclose a solid angle. The effect is a genuinely three slit effect, not decomposable in terms of two and one slit effects.
- Thus quantum theory contains effects which are not two slit separable because of multiparticle entanglement. Our three slit experiment involving the geometric phase brings out this point forcefully.
- The question of whether a single particle crossing a barrier with slits obeys two slit separability is ultimately an experimental one. The theoretical possibility of violations of two slit separability in such experiments was noted by Sorkin [R. Sorkin, Mod. Phys. Lett A 9, 3119 (1994)], who proposed that there may be theories going beyond quantum mechanics which admit such effects. There have been attempts [U. Sinha, et al., Science 329, 418 (2009)] to search for such effects in a three slit experiment using photons. Since these experiments are null experiments, one has to be careful to rule out all possible three slit effects that are present due to multiparticle entanglement.
- The experiment we propose here is just the simplest of a class of phenomena involving multiparticle entanglement, nonlocality and the geometric phase.