
Phys601: Problem set 2

Due date : 07 Apr 2012.

Topic : Probability distribution functions and their properties.

1. Tabulate the characteristic functions and first four moments for the probability distribution functions - Binomial, Multinomial, Poisson, Uniform, Normal, Exponential, χ^2 , Gamma, Lorentz and Cauchy, Log-normal distributions.
2. What will be the limit of Binomial distribution when the number of trials is large i.e. $n \rightarrow \infty$ and the probability of occurrence is small i.e., $p \rightarrow 0$ but $np = \lambda$ approaches a constant. Calculate the moments of the distribution. Give one practical example.
3. Show that the Binomial, Poisson and χ^2 distributions approach the Normal distribution in the limit of large number, mean value and many degrees of freedom respectively.
4. Give a proof of the central limit theorem for the case of independent but not identically distributed variables.
5. Let the variates x_i be exponentially distributed, the sample mean is given by

$$\bar{x} = \sum_i \frac{x_i}{N} \quad (1)$$

What would be the distribution of \bar{x} for small and large N .