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Matter effects in Atmospheric μ^-/μ^+ in Magnetized Iron Calorimeters

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The number of muons in an experiment is given by

$$N_\mu = \int dE \int d\cos\theta_n \left[\frac{\partial^2 \Phi_\mu}{\partial E \partial \cos\theta_n} P_{\mu\mu}(E,L) + \frac{\partial^2 \Phi_e}{\partial E \partial \cos\theta_n} P_{e\mu}(E,L) \right] \sigma_\mu(E) \qquad (1)$$

where E is the neutrino energy, θ_n is the nadir angle and L is the length of neutrino trajectory. Φ's are the neutrino fluxes and σ is the CC-DIS cross section. For upward going events, we can approximate $L \simeq 2R\cos\theta_n$, where R is the radius of earth. $P_{\mu\mu}$ is the muon neutrino survival probability, $P_{e\mu}$ is the probability of conversion of a ν_e to ν_μ. For anti-neutrinos, we have a similar equation

$$N_{\bar\mu} = \int dE \int d\cos\theta_n \left[\frac{\partial^2 \Phi_{\bar\mu}}{\partial E \partial \cos\theta_n} P_{\bar\mu\bar\mu}(E,L) + \frac{\partial^2 \Phi_{\bar e}}{\partial E \partial \cos\theta_n} P_{\bar e\bar\mu}(E,L) \right] \sigma_{\bar\mu}(E) \qquad (2)$$

In the absence of matter effects, we have $P_{\mu\mu} = P_{\bar\mu\bar\mu}$ and $P_{e\mu} = P_{\bar e\bar\mu}$ in the approximation where $\Delta_{21} \simeq \Delta_{sol}$ is neglected. In the multi-GeV range, the fluxes of neutrinos and anti-neutrinos are nearly equal. Hence in equations (1) and (2) the quantities in the square brackets are nearly equal. So the ratio $R_\mu = N_\mu/N_{\bar{\mu}}$ is essentially equal to $\sigma_\mu/\sigma_{\bar{\mu}} > 2$. We call this ratio R_0.

However, matter effects induce changes in all probabilities and depending on the sign of Δ_{31}, the ratio R_μ can be greater or less than R_0. We study the changes in R_μ due to matter effects for likely values of neutrino parameters.

First we study $P_{e\mu}$. In vacuum,

$$P_{e\mu} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(1.27\frac{\Delta_{31}L}{E}\right), \tag{3}$$

with $P_{\bar{e}\bar{\mu}} = P_{e\mu}$. Including matter effects modifies the above probability to

$$P^m_{e\mu} = \sin^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\left(1.27\frac{\Delta^m_{31}L}{E}\right), \tag{4}$$

where

$$\Delta^m_{31} = \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin 2\theta_{13})^2} \tag{5}$$

$$\sin 2\theta^m_{13} = \sin 2\theta_{13}\frac{\Delta_{31}}{\Delta^m_{31}} \tag{6}$$

where $A = 0.76 \times 10^{-4}\rho E$ is the Wolfenstein term, characterizing the matter effects. If Δ_{31} is positive, then we have $\Delta^m_{31} < \Delta_{31}$ and $P^m_{e\mu} > P_{e\mu}$ in the neighbourhood of the resonance. For anti-neutrinos $A \to -A$ and we have $P^m_{\bar{e}\bar{\mu}} < P_{\bar{e}\bar{\mu}}$ in the neighbourhood of the resonance. For negative Δ_{31} the situation is reversed. Note that, matter effects also affect muon and anti-muon survival probabilities in such a way that unitarity is preserved.

We wish to study the energy and trajectory length ranges where the matter effects are very large. The condition for resonance and the width of the resonance are given by

$$A = \Delta_{31}\cos 2\theta_{13} \tag{7}$$

$$\delta A = \Delta_{31}\sin 2\theta_{13} \tag{8}$$

For illustrative purposes, we choose $\sin^2 2\theta_{13} \simeq 0.1$. For $\Delta_{31} = 0.002$ eV^2, the resonance condition gives

$$\rho E \simeq 25 \pm 8. \tag{9}$$

In the mantle, the average density of matter is about 4.5 g/cm^3. Hence the resonance occurs for an energy of about 5.5 GeV. At this resonant energy $\sin^2 2\theta^m_{13} = 1.0$. For the energy range $3 - 7.5$ GeV, we have $\theta^m_{13} > \pi/4$ and $\sin^2 2\theta^m_{13} > 0.5$.

The increase in $P_{e\mu}$ due to the increase in θ_{13}^m is damped by the fact that Δ_{31}^m attains its minimum value $\Delta_{31} \sin 2\theta_{13}$ at the resonance energy. Hence, only very long pathlengths will it possible to have significant $\nu_e \to \nu_\mu$ oscillation probability.

$$0.5 \leq \sin^2 \left(1.27 \frac{\Delta_{31}^m L}{E}\right) \leq 1 \tag{10}$$

$$\pi/4 \leq 1.27 \frac{\Delta_{31} \sin 2\theta_{13} L}{E} \leq \pi/2 \tag{11}$$

From the above equation we find that for pathlengths in the range $6,000 - 12,000$ Km range, the path dependent phase term has a value greater than 0.5.

$P_{e\mu}$ is a product of three terms. First of these terms $\sin^2 \theta_{23} \simeq 0.5$ from the fit to atmospheric neutrino data. Without matter effects, the second term $\sin^2 2\theta_{13} = 0.1$. The pathlength dependent third term will average out to half for very long pathlengths. Hence we expect the average value of $P_{e\mu}$ to be about 0.025. Including the matter effects boosts this number by an order of magnitude. In the energy range of resonance width $(3 - 7.5)$ GeV the second term is always greater than 0.5 and for the range of pathlengths $(6,000 - 12,000)$ Km we have chosen, the third term is again always greater than 0.5. Conservatively we can expect the average value of the product of the second and third terms to be about $0.5(\simeq 0.75 \times 0.75)$. Thus including the matter effects, for positive Δ_{31}, boosts $\nu_e \to \nu_\mu$ oscillation by a factor of about 10. In the case of anti-neutrinos, the $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations are suppressed by a factor of about 4. This occurs because $\Delta_{31}^m \simeq 2\Delta_{31}$ near resonance. Hence the second term in $P_{\bar{e}\bar{\mu}}^m$ is smaller by a factor of 4 and the third term has an average value of 0.5.

We now turn to muon neutrino survival probability which has a more complicated matter dependence. In the limit $\Delta_{21} = 0$, we can choose vacuum eigenstates such that m_1 is essentially ν_e and m_2 has no ν_e component. Because of this reason, m_2 state is unaffected by inclusion of matter term. However, m_1 value rises with the Wolfenstein term and the $m_1 - m_2$ degeneracy of the vacuum is broken. The matter dependent mass-squared eigenvalues are given by

$$\begin{aligned} m_1^2(mat) &= \frac{\Delta_{31} + A - \Delta_{31}^m}{2} \\ m_2^2(mat) &= 0 \\ m_3^2(mat) &= \frac{\Delta_{31} + A + \Delta_{31}^m}{2} \end{aligned} \tag{12}$$

In vaccum, we have

$$P_{\mu\mu} = 1 - \cos^2 \theta_{13} \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta_{31} L}{E}\right)$$

$$- \sin^4\theta_{23}\sin^2 2\theta_{13}\sin^2\left(1.27\frac{\Delta_{31}L}{E}\right). \quad (13)$$

Including matter terms leads to

$$\begin{aligned} P^m_{\mu\mu} &= 1 - \sin^2 2\theta_{23}\cos^2\theta^m_{13}\sin^2\left(1.27\frac{(\Delta_{31}+A+\Delta^m_{31})L}{2E}\right) \\ &- \sin^4\theta_{23}\sin^2 2\theta^m_{13}\sin^2\left(1.27\frac{\Delta^m_{31}L}{E}\right) \\ &- \sin^2 2\theta_{23}\sin^2\theta^m_{13}\sin^2\left(1.27\frac{(\Delta_{31}+A-\Delta^m_{31})L}{2E}\right). \end{aligned} \quad (14)$$

In the above equation, the first two terms have their counterparts in the vacuum oscillation probability also. The third term is completely new and it arises because the matter term splits the degeneracy between m_1 and m_2.

Consider the difference

$$\begin{aligned} P_{\mu\mu} - P^m_{\mu\mu} &= 0.5(P^m_{e\mu} - P_{e\mu}) + \cos^2\theta^m_{13}\sin^2\left(1.27\frac{(\Delta_{31}+A+\Delta^m_{31})L}{2E}\right) \\ &\sin^2\theta^m_{13}\sin^2\left(1.27\frac{(\Delta_{31}+A+\Delta^m_{31})L}{2E}\right) \\ &- \cos^2\theta_{13}\sin^2\left(1.27\frac{\Delta_{31}L}{E}\right), \end{aligned} \quad (15)$$

where we have set $\theta_{23} = \pi/4$. We search for energy and pathlength ranges where this difference is large. We expect the energy range to be range of resonance, from the first term. This nearly true. The fourth term, which has a negative sign, is zero when the phase $(1.27\Delta_{31}L/E) \simeq n\pi$. Hence the above difference is likely to be large for those energies which are in the resonance energy range and for which the phase condition is satisfied.

We see from figure 1 that the maximum suppression of muon neutrino survival probability due to matter effects occurs when the values of L and E are such that $(1.27\Delta_{31}L/E) \simeq \pi$ and in all the cases the energy range of significant suppression has a large overlap with the resonance energy range. From figure 1, we note that observable matter effect occur for the energy range $5 - 10$ GeV and L range $6,000 - 9,700$ Km.

Using the probabilities of figure 1, we calculated the expected event rates for μ^- and μ^+ event rates in a magnetized iron calorimeter which is described in [1, 2]. We used Bartol flux tables [3], a modest muon identification efficiency of 50% and an

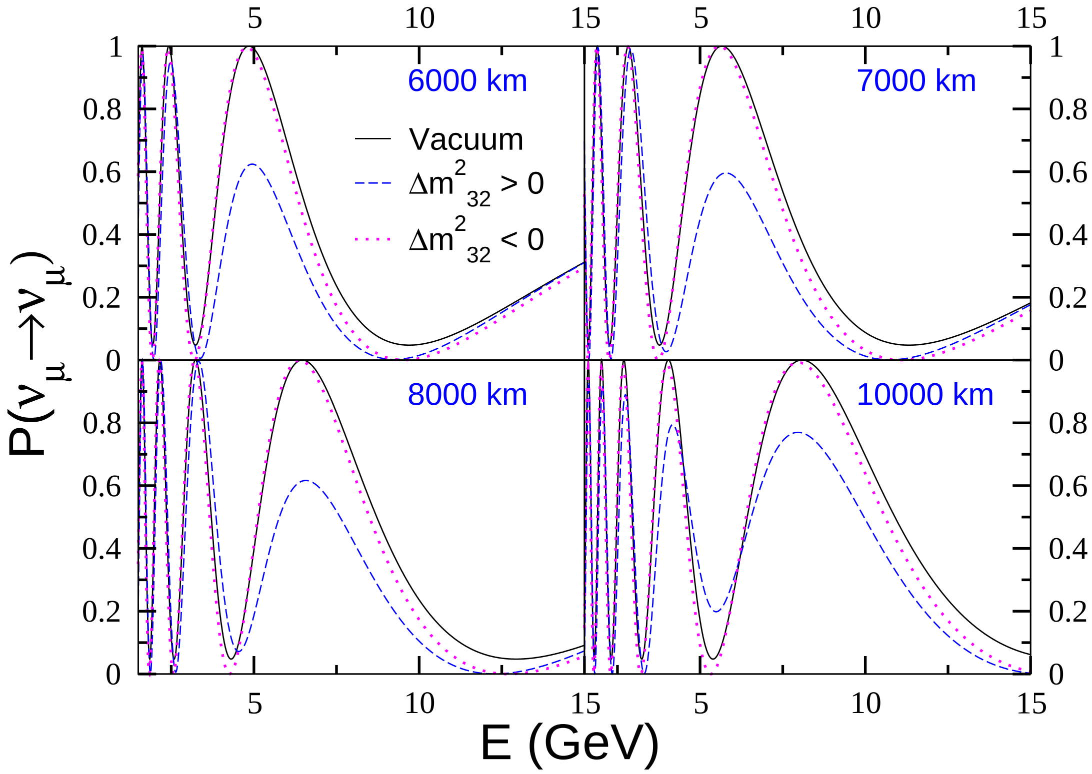

Figure 1: *The muon survival probability in vacuum and in matter for both signs of δm^2_{32} plotted against the neutrino energy for different values of baseline lengths, L (in km). The oscillation parameters used in all the plots are : $\delta m^2_{32} = 2 \times 10^{-3} eV^2$ and $\sin^2 2\theta_{23} = 1$.*

exposure time of 1000 Kton-Yr. The event rates are calculated for the energy range $5 - 10$ GeV and L range $6000 - 9700$ Km. The distribution of the event rates, both in the case of vacuum and matter oscillations are shown in figure 2 as function of L and in figure 3 as function of (L/E).

In both cases we notice that for the L range of $6000 - 9700$ Km, the matter dependent μ^- rate is noticeably smaller than vacuum rate, whereas the rates are indentical for μ^+. This conclusion holds for Δ_{31} positive. For negative Δ_{31}, vacuum and matter rates of μ^- coincide and matter rates of μ^+ show a deficit w.r.t. vacuum rates. The total number of μ^+ events, in the case of vacuum oscillations is 105 and it changes to 103 on inclusion of matter effects. The total number of μ^- events, in the

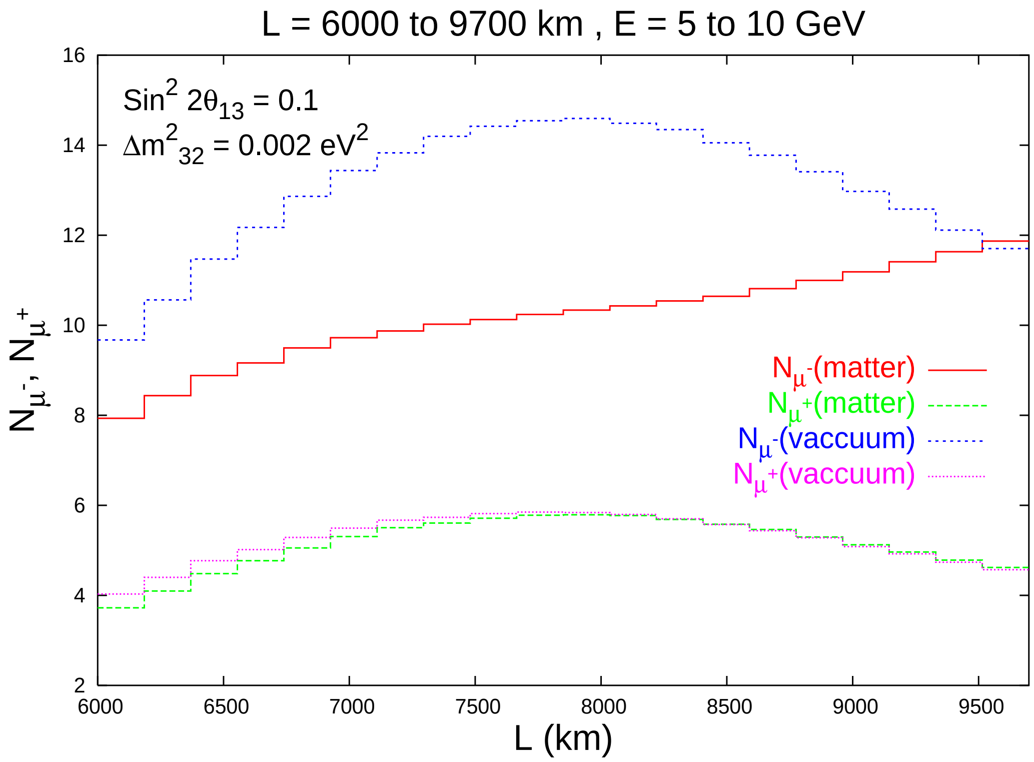

Figure 2: *The total event rate for muons and anti-muons in matter and in vacuum plotted against L for the restricted choice of L and E range.*

case of vacuum oscillations is 261 and this reduces to 204 on inclusion of matter effects. Thus we have a 4σ signal for matter effects for neutrino parameters $\Delta_{31} = 0.002$ eV^2 and $\sin^2 2\theta_{13} = 0.1$. We estimate that the matter effect will lead to about 2.5σ signal for the same Δ_{31} and $\sin^2 2\theta_{13} = 0.05$. A systematic study of sensitivity of magnetized iron detectors to matter effects as function of θ_{13} and Δ_{31} is underway.

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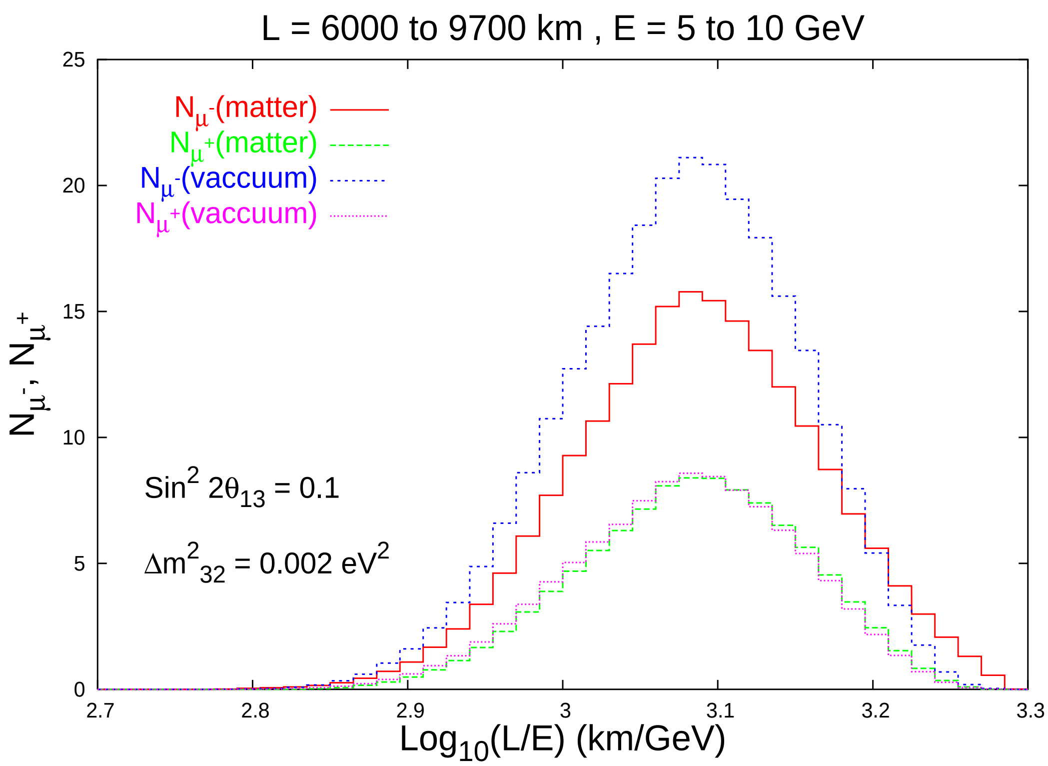

Figure 3: *The total event rate for muons and anti-muons in matter and in vacuum plotted against* $Log_{10}(L/E)$ *for the restricted choice of L and E range.*

References

[1] N. Y. Agafonova *et al.* [MONOLITH Collaboration], see http://castore.mi.infn.it/∼monolith/

[2] See http://www.imsc.res.in/∼ino; and working reports and talks therein.

[3] V. Agrawal, T. K. Gaisser, P. Lipari and T. Stanev, Phys. Rev. D **53**, 1314 (1996)