Atmospheric $\nu$ as a Probe of CPT Violation

(based on hep-ph/0312027)

Poonam Mehta

Department of Physics and Astrophysics, University of Delhi, India
work done with A. Datta, R. Gandhi and S. Uma Sankar
Outline of the talk . . .

- General motivation for CPT Violation
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
- Survival Probability - 2 flavour case
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
- Survival Probability - 2 flavour case
- Observables to look for CPTV
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
- Survival Probability - 2 flavour case
- Observables to look for CPTV
- Bounds of CPTV
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
- Survival Probability - 2 flavour case
- Observables to look for CPTV
- Bounds of CPTV
- Results
Outline of the talk . . .

- General motivation for CPT Violation
- CPT Violation in neutrino oscillations
- Survival Probability - 2 flavour case
- Observables to look for CPTV
- Bounds of CPTV
- Results
- Conclusions
The CPT theorem ⇒ Invariance of $\mathcal{L}(x)$ under action of C, P and T

CPT ⇒ property of any QFT in FLAT space time which respects:

(i) Locality

(ii) Unitarity

(iii) Lorentz Symmetry

$$\Theta \mathcal{L}(x) \Theta^\dagger = \mathcal{L}(-x) , \quad \Theta = CPT , \quad \mathcal{L} = \mathcal{L}^\dagger$$


W. Pauli, Neils Bohr and the Development of Physics, Mc Graw-Hill, New York, 1955,

p. 30
CPT Invariance and Lorentz symmetry are pillars of modern physics. Tests of fundamental principles of invariance are important due to the far-reaching consequences of their violations.
CPT Invariance and Lorentz symmetry are pillars of modern physics. Tests of fundamental principles of invariance are important due to the far-reaching consequences of their violations.

String or other unified theories can induce small violations of CPT and Lorentz symmetry into the SM at low energies naturally, which can be tested at levels reachable by high precision experiments.

General Motivation for CPTV...

**CPT** Invariance and Lorentz symmetry are pillars of modern physics. Tests of fundamental principles of invariance are important due to the far-reaching consequences of their violations.

String or other unified theories can INDUCE small violations of CPT and Lorentz symmetry into the SM at low energies naturally, which can be tested at levels reachable by high precision experiments.

- D. Colladay and V. A. Kostelecky, PRD 55, 6760 (1997); PRD 58, 116002 (1998)

CPTV implies LV but not vice-versa...

AN ARTIST'S IMPRESSION OF SPACE-TIME FOAM

AFTER WEINBERG 99

N. E. Mavromatos, hep-ph/0402005
General Motivation for CPTV . . .

SPACE–TIME FOAMY SITUATIONS
NON UNITARY (CPT VIOLATING) EVOLUTION
OF PURE STATES TO MIXED ONES

| ... >

modified temporal evolution of $\rho$:

$$\frac{d}{dt} \rho = i [\rho, H] + \Delta H(\rho) \rho$$

quantum mechanical terms

quantum mechanics

violating term

$\rho_{out} = \text{density matrix}$

$= \text{Tr}_{\text{unobs}} |\psi><\psi|$
It is crucial to test whether the “foundations” on which the theory is based are indeed firm and unshakable.
It is crucial to test whether the “foundations” on which the theory is based are indeed firm and unshakable.

Phenomenologically, we are interested in the effective low-energy violation of CPT, i.e., the complete fundamental theory may respect CPT invariance.
It is crucial to test whether the “foundations” on which the theory is based are indeed firm and unshakable.

Phenomenologically, we are interested in the effective low-energy violation of CPT, i.e., the complete fundamental theory may respect CPT invariance.

So far, Neutrino factories (NF) have been considered to set significant bounds on CPTV parameters, but NF are far in future!

V. Barger et. al., PRL 85, 5055 (2000)
Different masses for neutrinos and antineutrinos was postulated as a potential solution to the LSND anomaly in Murayama+Yanagida (PLB 520, 263 (2001)) and further pursued by Barenboim et al.

Barenboim et al., hep-ph/0212116
Different masses for neutrinos and antineutrinos was postulated as a potential solution to the LSND anomaly in Murayama+Yanagida (PLB 520, 263 (2001)) and further pursued by Barenboim et al.

Currently this form of CPTV solution to $\nu$ puzzles plus LSND is experimentally disfavoured.

Barenboim et al., hep-ph/0212116
ν OSCILLATIONS ARE SENSITIVE TO VIOLATION OF DISCRETE SYMMETRIES: CP, T, CPT

\[ P(\nu_\alpha \rightarrow \nu_\beta) \xleftarrow{CP} \quad \xrightarrow{T} \quad P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \xleftarrow{CPT} \quad \xrightarrow{T} \quad P(\nu_\beta \rightarrow \nu_\alpha) \xleftarrow{CP} \quad \xrightarrow{T} \quad P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \]
Violation of discrete symmetries . . .

CP is violated when

\[ P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta), \quad \beta \neq \alpha \]
Violation of discrete symmetries . . .

- **CP** is violated when
  \[ P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \quad \beta \neq \alpha \]

- **T** is violated when
  \[ P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha), \quad \beta \neq \alpha \]
Violation of discrete symmetries . . .

- CP is violated when
  \[ P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \quad \beta \neq \alpha \]

- T is violated when
  \[ P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha), \quad \beta \neq \alpha \]

- CPT is violated when either
  \[ P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\beta \to \bar{\nu}_\alpha), \quad \beta \neq \alpha \]
  or,
  \[ P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) \]
Violation of discrete symmetries . . .

CP is violated when

\[ P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta), \quad \beta \neq \alpha \]

T is violated when

\[ P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\nu_\beta \rightarrow \nu_\alpha), \quad \beta \neq \alpha \]

CPT is violated when either

\[ P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha), \quad \beta \neq \alpha \]

or,

\[ P(\nu_\alpha \rightarrow \nu_\alpha) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \]

Also, MATTER EFFECTS ➢ apparent (extrinsic) CP & CPT violation even if mass matrix is CP conserving
We consider effective C- & CPT-odd interaction terms
\[ \mathcal{L}_{\text{eff}} = \bar{\nu}_{L}^{\alpha} b_{\alpha\beta}^{\mu} \gamma_{\mu} \nu_{L}^{\beta}, \quad \alpha, \beta \rightarrow \text{flavour indices} \]
We consider effective C- & CPT-odd interaction terms

\[ \mathcal{L}_{\text{eff}} = \bar{\nu}_L^\alpha b_{\alpha \beta}^\mu \gamma_\mu \nu_L^\beta, \quad \alpha, \beta \rightarrow \text{flavour indices} \]

**DISPERSION RELATION** in ultra-relativistic limit,

\[ A = \frac{m^2}{2p} \pm b \]

Coleman & Glashow, PRD 59, 116008 (1999); Pakvasa, hep-ph/0110175
We consider effective C- & CPT-odd interaction terms

\[ \mathcal{L}_{\text{eff}} = \bar{\nu}_L^\alpha b_{\alpha\beta}^\mu \gamma_\mu \nu_L^\beta, \quad \alpha, \beta \rightarrow \text{flavour indices} \]

**DISPERSION RELATION** in ultra-relativistic limit,

\[ A = \frac{m^2}{2p} \pm b \]

Coleman & Glashow, PRD 59, 116008 (1999); Pakvasa, hep-ph/0110175

\[ m^2 \equiv mm^\dagger \text{ is the Hermitian mass squared matrix} \]
We consider effective C- & CPT-odd interaction terms
\[ \mathcal{L}_{\text{eff}} = \bar{\nu}_L^\alpha b_{\alpha\beta}^{\mu} \gamma_{\mu} \nu_L^\beta, \quad \alpha, \beta \rightarrow \text{flavour indices} \]

**DISPERSION RELATION** in ultra-relativistic limit,
\[ A = \frac{m^2}{2p} \pm b \]

Coleman & Glashow, PRD 59, 116008 (1999); Pakvasa, hep-ph/0110175

\[ m^2 \equiv \mm^\dagger \] is the Hermitian mass squared matrix
\[ b \equiv b^0 \] is Non-diagonal matrix in flavour basis
➢ For neutrinos:

\[ P_{\alpha\alpha}(L) = 1 - \sin^2 2\Theta \sin^2(\Delta L/4) \]
For neutrinos:

\[ P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2(\Delta L/4) \]

For anti-neutrinos:

\[ P_{\alpha\alpha}(L) = 1 - \sin^2 2\bar{\theta} \sin^2(\bar{\Delta} L/4) \]
Survival Probability – 2 flavour case . . .

➢ For neutrinos:

\[
P_{\alpha\alpha}(L) = 1 - \sin^2 2\Theta \sin^2(\Delta L/4)
\]

➢ For anti-neutrinos:

\[
P_{\bar{\alpha}\bar{\alpha}}(L) = 1 - \sin^2 2\bar{\Theta} \sin^2(\bar{\Delta} L/4)
\]

where,

\[
\Delta \sin 2\Theta = |(\delta m^2 / E) \sin 2\theta_m + 2\delta b e^{i\eta_1} \sin 2\theta_b|
\]

\[
\Delta \cos 2\Theta = (\delta m^2 / E) \cos 2\theta_m + 2\delta b e^{i\eta_1} \cos 2\theta_b
\]

\[
\bar{\Delta} \sin 2\bar{\Theta} = |(\delta m^2 / E) \sin 2\theta_m - 2\delta b e^{i\eta_1} \sin 2\theta_b|
\]

\[
\bar{\Delta} \cos 2\bar{\Theta} = (\delta m^2 / E) \cos 2\theta_m - 2\delta b e^{i\eta_1} \cos 2\theta_b
\]

The effective mixing angle \( \Theta \) and oscillation wavelength \( \Delta \) can vary dramatically with \( E \) for appropriate values of \( \delta b \).
Assume that $m^2$ & $b$ are diagonalized by same angle:
- set $\theta_m = \theta_b = \theta$,

Take the complex phase to be zero:
- set $\eta_1 = 0$,

We have used equal masses for $\nu$ and $\bar{\nu}$:
- set $m_\nu = m_{\bar{\nu}} = m$
For simple $2f$ case:

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left( \left( \frac{\delta m^2}{4E} \pm \frac{\delta b}{2} \right) L \right)$$

Note: E-independent contribution to the oscillation wavelength.

In proper units,

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left[ 1.267 \times (\delta m^2 \pm 10^{-18} \times 2\delta bE) \frac{L}{E} \right]$$

Probability Difference:

$$\Delta P_{\alpha\alpha}^{\text{CPT}} = P_{\alpha\alpha} - P_{\tilde{\alpha}\tilde{\alpha}}$$

$$\Delta P_{\alpha\alpha}^{\text{CPT}} = - \sin^2 2\theta \sin \left( \frac{\delta m^2 L}{2E} \right) \sin(\delta b L)$$
An important consequence of Violation of CPT symmetry is that it leads to a modified dispersion relation.
Few comments are in order . . .

- An important consequence of Violation of CPT symmetry is that it leads to a modified dispersion relation.

- Modified dispersion relations for neutrino lead to deviations from the characteristic L/E-oscillatory behaviour, which means that precision oscillation measurements can set unprecedented bounds on such effects!
Observables for CPTV . . .

- Observable CPTV in 2f case ➔
  consequence of interference of the CPT-EVEN AND CPT-ODD terms
• Observable CPTV in 2f case ⇒
consequence of interference of the CPT-EVEN AND CPT-ODD terms

• Compare $\nu_\mu \rightarrow \nu_\mu$ vs $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$
  
  CPT ODD oscillation argument $\sim L$ while,
  CPT EVEN oscillation argument $\sim L/E$
Observables for CPTV . . .

- Observable CPTV in 2f case ⇒
consequence of interference of the CPT-EVEN AND CPT-ODD terms

- Compare $\nu_\mu \rightarrow \nu_\mu$ vs $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$

  CPT ODD oscillation argument $\sim L$ while,
  CPT EVEN oscillation argument $\sim L/E$

- Look at Ratio of muon to anti-muon events :

  $\frac{N(\nu_\mu \rightarrow \nu_\mu)}{N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)}$ vs $L, L/E$
The number of muon events:

\[ N = N_n \times M_d \int \sigma_{\nu_{\mu}N}^{CC} P(\nu_{\mu} \rightarrow \nu_{\mu}) \frac{dN_{\nu}}{dE_{\nu}} \ dE_{\nu} \]

- \( N_n = 6.023 \times 10^{32} \)
- \( M_d \rightarrow \) detector mass (in kT)
- For \( E > 1.8 \text{ GeV} \) ➞ \( \sigma_{\nu_{\mu}N}^{CC} \rightarrow \) DIS X-section
- For \( E < 1.8 \text{ GeV} \) ➞ \( \sigma_{\nu_{\mu}N}^{CC} \rightarrow \) QE X-section
- Bartol Atmospheric flux is used.
- Muon detection threshold, \( E_{th} = 1 \text{ GeV} \).
We adopt 2 flavour approximation for atmospheric $\nu$ for studying the $b$ matrix:

- Assume $\theta_{13}$ → small (below CHOOZ bound)
- Assume $\delta m_{21}^2$ → small (compared to $\delta m_{32}^2$)

Thus, Matter and related three flavour effects can be safely neglected.

Also, matter effects show up in atmospheric neutrinos for $\sin^2 2\theta_{13} \sim 0.1$ and baselines above 7000 km.

Banuls et. al., PLB 513, 391 (2001)

Matter effects are very small for the case of survival probabilities...
What should be the detector type?

- Our Aim: Testing CPTV using atmospheric $\nu$
- Main Requirement: Charge Identification of muons
- Detector type: Large mass magnetized iron calorimeter
  - Examples of such a detector:
    - MONOLITH: was initially proposed for Gran Sasso
      MONOLITH proposal, http://castore.mi.infn.it/~monolith
    - INO: currently planned for location in India
      http://www.imsc.res.in/~ino
Characteristics of such a detector . . .

➢ Magnetized iron (50-100 kT) with RPC or glass spark chambers
➢ Excellent muon track and charge discrimination with 5% energy resolution
➢ Lower threshold possible, $E_{th} \approx 1 - 2\, GeV$
➢ Backgrounds due to pion and kaon decays can be greatly reduced by kinematic cuts
➢ **SUPERIOR L/E RANGE** as high density contains many HE events
Physics capabilities of such a detector . . .

➢ Atmospheric neutrinos: Clean detection of L/E dependance of event rate, providing unambiguous signature of oscillations.
➢ As an end detector for neutrino beams from $\beta$-beam set-up: to study the oscillation channels, $\nu_e \rightarrow \nu_\mu$ or $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$
➢ As an end detector for neutrino beams from neutrino factory set-up: $\nu_\mu (\bar{\nu}_\mu)$ and $\bar{\nu}_e (\nu_e)$ from $\mu^- (\mu^+)$ decay
  • $\theta_{13}$
  • Sign of $\delta m_{32}^2$
  • $\delta_{CP}$
➢ Detection of HE cosmic-ray muons in the PeV range
Atmospheric neutrinos . . .

➢ Advantage: The disappearance probability can be measured with single detector and 2 equal sources.

➢ Main goal: Study oscillation pattern in atmospheric neutrino events.

\[ A_{cr} + A_{air} \rightarrow \pi^+ + \ldots \]

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

\[ \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \]

\[ \frac{N_{up}(L/E)}{N_{down}(L/E)} \sim P_{\mu\mu} \]

TIFR, July 20, 2004 – p.22/29
<table>
<thead>
<tr>
<th>EXPER.</th>
<th>SECTOR</th>
<th>PARAMS. (J=X,Y)</th>
<th>BOUND (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penning Trap</td>
<td>electron</td>
<td>$\bar{b}_J^e$</td>
<td>$5 \times 10^{-25}$</td>
</tr>
<tr>
<td>Hg–Cs clock comparison</td>
<td>electron</td>
<td>$\bar{b}_J^e$</td>
<td>$10^{-27}$</td>
</tr>
<tr>
<td></td>
<td>proton</td>
<td>$\bar{b}_J^p$</td>
<td>$10^{-27}$</td>
</tr>
<tr>
<td></td>
<td>neutron</td>
<td>$\bar{b}_J^n$</td>
<td>$10^{-30}$</td>
</tr>
<tr>
<td>H Maser</td>
<td>electron</td>
<td>$\bar{b}_J^e$</td>
<td>$10^{-27}$</td>
</tr>
<tr>
<td></td>
<td>proton</td>
<td>$\bar{b}_J^p$</td>
<td>$10^{-27}$</td>
</tr>
<tr>
<td>spin polarized matter</td>
<td>electron</td>
<td>$\bar{b}_J^e / b^e_Z$</td>
<td>$10^{-29} / 10^{-28}$</td>
</tr>
<tr>
<td>He–Xe Maser</td>
<td>neutron</td>
<td>$\bar{b}_J^n$</td>
<td>$10^{-31}$</td>
</tr>
<tr>
<td>Muon g–2</td>
<td>muon</td>
<td>$\bar{b}_J^\mu$</td>
<td>$2 \times 10^{-23}$</td>
</tr>
</tbody>
</table>

X,Y,Z celestial equatorial coordinates: $\bar{b}_J = b_3 - m_d 30 - H_{12}$

(Bluhm, hep–ph/0111323)
We show the plots of ratio of total (up+down) muon survival events to those of anti-muons vs $L$ & $L/E$ for different values of $\delta b$:

**The main features are:**

(i) Solid red line $\implies$ CPT conserving ($\delta b = 0$) case:
Shape & position is dominated by $\sigma_\nu/\sigma_\bar{\nu}$ at relatively low (few GeV) $E$
Small wiggles are due to the difference in fluxes of $\nu$ & $\bar{\nu}$ at different $E$ and $L$

(ii) CPT odd ($\delta b \neq 0$) case:
Main features can be extracted by looking at the probability difference:

$$\Delta P_{\mu\mu}^{\text{CPT}} = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}} = -\sin^2 2\theta \sin \left( \frac{\delta m^2 L}{2E} \right) \sin(\delta b L)$$
Results . . .

For $\delta b \neq 0$, the main features are:

- **NODES** (i.e. intersection of $\delta b \neq 0$ curves with the $\delta b = 0$ curve) whenever the CPTV Probability difference $\rightarrow 0$.

- **Nodes in plot vs L:**
  $$\sin(\delta b L) = 0$$
  $$L (\text{km}) = n\pi/\delta b (\text{GeV})$$

  ↓ sensitive to magnitude of $\delta b$

- **Nodes in plot vs L/E:**
  $$\sin(\delta m^2 L/2E) = 0$$
  Common Node

  ↓ not sensitive to $\delta b$

- **DIPS near** $\log(L/E) \approx 2.5 \text{ km/GeV}$ or $L/E \approx 310 \text{ km/GeV}$ for $\delta b \leq 10^{-22}$

  GeV. This is explained by the fact that for this value $\delta m^2 L/4E = \pi/4$. 

TIFR, July 20, 2004 – p.25/29
Results for $\delta b$ . . .

NODES: $\sin(\delta b L) = 0 \implies L(km) = n\pi/\delta b(GeV)$
Results for $\delta b$ . . .

NODES: $\sin(\delta m^2 L/2E) = 0 \Rightarrow \text{COMMON NODE}

\begin{align*}
\delta b = 0 & \quad \delta b = 1 \times 10^{-21} \\
\delta b = 3 \times 10^{-23} & \quad \delta b = 1 \times 10^{-22}
\end{align*}
Comparing bounds on CPTV . . .

- NF bounds on $\delta b$
  $\delta b > 3 \times 10^{-23}$ GeV
  V. Barger et. al., PRL 85, 5055 (2000)

- Our bounds on $\delta b$
  ➔ The L/E dependance of ratio of muon to anti-muon events shows sensitivity to the presence of CPTV for
  $\delta b > 3 \times 10^{-23}$ GeV
  ➔ The L dependance of the ratio of muon to anti-muon events is sensitive to detecting both the presence and magnitude of CPTV for
  $\delta b > 3 \times 10^{-22}$ GeV
Atmospheric neutrinos & A large mass iron calorimeter (for e.g. Indian Neutrino Observatory) can allow us to set significant bounds on all types of CPTV in the neutrino sector.
Atmospheric neutrinos & A large mass iron calorimeter (for e.g. Indian Neutrino Observatory) can allow us to set significant bounds on all types of CPTV in the neutrino sector.

The presence of CPTV can be detected by looking at the ratio

\[ \frac{N(\nu_\mu \rightarrow \nu_\mu)}{N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)} \]

vs L, L/E for \( \delta b > 3 \times 10^{-23} GeV. \)
Atmospheric neutrinos & A large mass iron calorimeter (for e.g. Indian Neutrino Observatory) can allow us to set significant bounds on all types of CPTV in the neutrino sector.

The presence of CPTV can be detected by looking at the ratio

\[ \frac{N(\nu_\mu \rightarrow \nu_\mu)}{N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)} \]

vs L, L/E for \( \delta b > 3 \times 10^{-23} \text{GeV} \).

The measure of the magnitude of CPTV can be also be possibly obtained by studying the position of zeros and minimas arising in plots of

\[ \frac{N(\nu_\mu \rightarrow \nu_\mu)}{N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)} \]

vs L for somewhat higher values of these parameters.