Introduction to Numerical Analysis with Python

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A Note to the Reader

This Manual was written for use in the Numerical Analysis Lab of MSc Physics course at Dept. of Physics & Astrophysics, University of Delhi. However, it can be used at the undergraduate level too or in other courses since it does not assume any knowledge of either Numerical Analysis or Python.

Like any lab manual, it is not a place where you can learn the theory of Numerical Analytic methods, though at various places we have tried to give a brief introduction to them. Instead, it is primarily a Manual focussed on using the numerical methods in programming in Python.

The Manual is organized into 13 Chapters. The first three chapters are about Python language and the others discuss various topics in Numerical Programming.

After an introductory chapter with a brief introduction to algorithms and computer languages as well some very basic Python commands, the second chapter discusses Python language in somewhat more detail. This includes Python variables, operations with variable, conditional statements and iteration, python arrays, user defined functions and manipulating data files. The idea behind this is not to provide exhaustive details of the language but instead to serve as an introduction which allows the reader to explore further. Thus, apart from describing the various structures and constructs in Python, there are several sample programs which illustrate the use of these structures. There are also exercises in programming after each chapter which the reader should do.

Chapter 3 is an introduction to graphics in Python. We discuss various options available for plotting with Matplotlib library of routines.

The rest of the chapters are basically the use of Python to solve various Numerical problems using Numerical analysis. The techniques discussed include summing of finite and infinite series, finding roots of a function, solving ordinary differential equations, integration of functions and matrices. We also discuss numerical solutions to time independent Schrodinger equation, interpolation and regression. There are two introductory chapters on Monte Carlo methods as used for integration and simulation.

Each chapter follows the same format of briefly describing the numerical method and then using it to show how programs in Python can implement that method.

Programming, like swimming or cycling can only be learnt by actually doing it. Python is a very exhaustive language which is very versatile. In addition, there are many structures and options in

Python which we have not listed. The reader is encouraged to use the vast resources on the Internet on Python to explore these. **The programs described in the book are not necessarily written in the most efficient way nor using the full power of Python. Instead the idea is to convey the logic in the simplest possible way and encourage the reader to modify the programs using other structures and learn about them.** The exercises are an essential component of the Manual and the reader is advised to attempt all of them in the Manual.

We would very much like to get your suggestions regarding how to improve this Manual. In addition, if there are any errors or misprints that are spotted in the Manual, we would like to hear from you. **Please send a mail with the suggestions/errors etc. to shobhit.mahajan@gmail.com making sure you quote the version number of the Manual as well as the Modification date of the Manual you are using. The version number and date are on the title page of the Manual.**

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